407(5a): The Effect of Gravitation on the H atom

The Hamiltonian is:

$$H = \frac{p^2}{2m} - \frac{e^2}{4\pi\epsilon_0 r} \quad -(1)$$

and its expectation value is:

$$\langle H \rangle = \left\langle \frac{p^2}{2m} \right\rangle - \left\langle \frac{e^2}{4\pi\epsilon_0 r} \right\rangle \quad -(2)$$

$$= \frac{1}{2}\left(\frac{\alpha}{n}\right)^2 mc^2 - \left(\frac{\alpha}{n}\right)^2 mc^2$$

$$= -\frac{1}{2}\left(\frac{\alpha}{n}\right)^2 mc^2$$

where

$$\frac{1}{2}\frac{v^2}{c^2} = \frac{1}{2}\left(\frac{\alpha}{n}\right)^2 \quad -(3)$$

is the Thomas half. In the de Sitter theory the effect of gravitation is:

$$v^2 \to v^2 + \frac{2mG}{r} \quad -(4)$$

on the classical level. On the quantum levels the effect of gravitation on $\langle H \rangle$ is to change it to:

$$\langle H \rangle = -\frac{1}{2} m \langle v^2 \rangle - mMG \left\langle \frac{1}{r} \right\rangle \quad -(5)$$

Using

$$\left\langle \frac{e^2}{4\pi\epsilon_0 r} \right\rangle = \frac{\alpha^2}{n^2} mc^2 \quad -(6)$$

it is found that

$$\left\langle \frac{1}{r} \right\rangle = \frac{1}{n^2 a_0} \quad -(7)$$

where a_0 is the Bohr radius.

So the energy levels of the H atom are changed to:

$$\boxed{E = \langle H_0 \rangle = -\frac{1}{2}\left(\frac{\alpha}{n}\right)^2 mc^2 - \frac{mMG}{n^2 a_0}} \quad -(8)$$

) However, the result (5) is obtained much more simply by realizing that:

$$U_g = -\frac{mMG}{r} \quad -(9)$$

the classical gravitational potential. The de Sitter theory is based on the incorrect Schwarzschild metric, a solution of the Einstein field equation obtained by neglecting torsion. The effect of gravitation from the second term on the right hand side of eq. (8) is entirely negligible for the electron proton system, because

$$m_e = 9.10953 \times 10^{-31} \text{ kg}$$
$$m_p = 1.67265 \times 10^{-27} \text{ kg}$$
$$G = 6.67 \times 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2}$$
$$a_0 = 5.29177 \times 10^{-11} \text{ m}$$

It is possible to try a theory in which the electron of mass m interacts with a very large mass M such as that of the Earth. In this case:

$$E = \langle H_0 \rangle = -\frac{1}{2}\left(\frac{\alpha}{n}\right)^2 mc^2 - \frac{mMG}{r} \quad -(9)$$

where

$$m = 9.10953 \times 10^{-31} \text{ kg} \quad -(10)$$
$$M = 5.98 \times 10^{24} \text{ kg}$$
$$r = 6.378 \times 10^{9} \text{ m}$$

so for $n = 1$:

$$E_0 = -\alpha^2 mc^2 = -2.185 \times 10^{-18} \text{ J} \quad -(11)$$
$$E_G = -\frac{mMG}{r} = -1.0756 \times 10^{-25} \text{ J}$$

If the proton's gravitational attraction to the earth is considered then E_G increases by a factor of 1836, so

$$E_G = -1.974 \times 10^{-22} \quad -(12)$$

However, the macroscopic gravitational energy does not [illegible] the spectral detail, because the latter depends [illegible] the difference of energy levels. To observe gravitational effects [illegible] method is needed to observe absolute energy levels. This can be achieved with optical pumping and double resonance.

However, the de Sitter theory is trivially [illegible] by considering the classical kinetic energy:

$$T = \frac{1}{2} m v^2 \quad - (13)$$

This familiar quantity can be expressed in terms of the Thomas half:

$$T = \frac{1}{2} \frac{v^2}{c^2} mc^2 \quad - (14)$$

The de Sitter theory asserts that:

$$v^2 \rightarrow v^2 + \frac{2MG}{r} \quad - (15)$$

in the presence of gravitation, so according to de Sitter the hamiltonian is:

$$H = \frac{1}{2} \frac{1}{c^2} \left(v^2 + \frac{2MG}{r} \right) mc^2$$

$$= \frac{1}{2} m v^2 + \frac{mMG}{r} \quad - (16)$$

and there is no gravitational attraction because the sign of the gravitational potential is positive:

$$U = ? \frac{mMG}{r} \quad - (17)$$

Gravitational attraction requires:

$$U = - \frac{mMG}{r} \quad - (18)$$

as is well known.

The origin of the problem (17) is the so called

r) Schwarzschild metric

$$ds^2 = c^2 d\tau^2 = \left(1 - \frac{2mG}{rc^2}\right) c^2 dt^2 - \left(1 - \frac{2mG}{rc^2}\right)^{-1} dr^2 - r^2 d\phi^2 \quad -(19)$$

As shown in Note 405(3), rotation with

$$\phi' = \phi + \omega t \quad -(20)$$

produces the de Sitter precession:

$$\Delta \phi_g = 2\pi \left(\left(1 - \frac{v_1^2}{c^2}\right)^{-1/2} - 1 \right) \quad -(21)$$

where

$$v_1^2 = v^2 + \frac{2mG}{r} \quad -(22)$$

In the limit:

$$v_1 << 1 \quad -(23)$$

eq. (15) is obtained, Q.E.D.

The positive sign of $\frac{2mG}{r}$ comes from the negative sign of $\frac{2mG}{rc^2}$ in the Schwarzschild metric.

For the first time, it is revealed that the Schwarzschild metric is incorrect at the basic level.