84(3): Antisymmetry and Forward Precession

· For both forward and retrograde precession the lagrangian is:

$$\mathcal{L} = -\frac{mc^2}{\gamma} - U \quad -(1)$$

$$= -mc^2\left(1 - \frac{\dot{\underline{r}}\cdot\dot{\underline{r}}}{c^2}\right)^{1/2} + \frac{mMG}{r}$$

The Euler Lagrange equation is:

$$\frac{d\underline{p}}{dt} = \frac{\partial\mathcal{L}}{\partial\underline{r}} = \frac{d}{dt}\frac{\partial\mathcal{L}}{\partial\dot{\underline{r}}}. \quad -(2)$$

From eqns. (1) and (2)

$$\ddot{\underline{r}} = -\frac{MG}{\gamma^3}\frac{\underline{r}}{r^3} \quad -(3)$$

using:

$$\underline{p} = \gamma m\dot{\underline{r}} \quad -(4)$$

Here:

$$\gamma = \left(1 - \frac{\dot{\underline{r}}\cdot\dot{\underline{r}}}{c^2}\right)^{-1/2} \quad -(5)$$

Eq. (3) gives retrograde precession using $\underline{r}$ as the ...nges Lagrange variable. In the previous two notes it ...s been shown that antisymmetry is obeyed by retrograde ...recession.

The same lagrangian (1) gives forward precession ...f it is written in terms of its X and Y ...omponents:

$$\mathcal{L} = -mc^2\left(1 - \frac{\dot{X}^2 + \dot{Y}^2}{c^2}\right)^{1/2} + \frac{mMG}{(X^2+Y^2)^{1/2}} \quad -(6)$$

If the proper Lagrange variables are chosen to be X and Y. It follows that there are two Euler Lagrange equations:

$$\frac{\partial \mathcal{L}}{\partial X} = \frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{X}}\right) \quad -(7)$$

$$\frac{\partial \mathcal{L}}{\partial Y} = \frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{Y}}\right) \quad -(8)$$

The lagrangian for forward precession is given by eq. (1). The forward precession must also obey antisymmetry.

As shown in previous work eqns. (7) and (8) imply:

$$\underline{g} = \underline{\ddot{r}} = \frac{MG}{\gamma r^3 c^2}\underline{\dot{r}}\left(\underline{\dot{r}} \cdot \underline{r}\right) - \frac{MG}{\gamma}\frac{\underline{r}}{r^3} \quad -(9)$$

In the first approximation assume the Newtonian limit, so:

$$\underline{\Omega} = -\frac{MG}{c}\frac{\underline{r}}{r^2} \quad -(10)$$

with:

$$\underline{g} = -\omega_0 \underline{\Omega}. \quad -(11)$$

$$-(12)$$

It follows that:

$$\underline{g} = \frac{MG}{\gamma r^3 c^2}\underline{\dot{r}}\left(\underline{\dot{r}} \cdot \underline{r}\right) - \frac{MG}{\gamma}\frac{\underline{r}}{r^3} = \omega_0\frac{MG}{c}\frac{\underline{r}}{r^2}$$

i.e.

$$\frac{\underline{\dot{r}}\left(\underline{\dot{r}} \cdot \underline{r}\right)}{\gamma r c^2} - \frac{1}{\gamma}\frac{\underline{r}}{r} = \frac{\omega_0}{c}\underline{r} \quad -(13)$$

In component form:

$$\frac{\dot{x}\,(\dot{x}x+\dot{y}y)}{\gamma c^2(x^2+y^2)^{1/2}} - \frac{1}{\gamma}\frac{x}{(x^2+y^2)^{1/2}} = \frac{\omega_0}{c}x \quad -(14)$$

$$\frac{\dot{y}\,(\dot{x}x+\dot{y}y)}{\gamma c^2(x^2+y^2)^{1/2}} - \frac{1}{\gamma}\frac{y}{(x^2+y^2)^{1/2}} = \frac{\omega_0}{c}y \quad -(15)$$

i.e.
$$\frac{\omega_0}{c} = \frac{\dot{x}}{\gamma c^2 x}\frac{(x\dot{x}+\dot{y}y)}{(x^2+y^2)^{1/2}} - \frac{1}{\gamma(x^2+y^2)^{1/2}} \quad -(16)$$

$$= \frac{\dot{y}}{\gamma c^2 y}\frac{(\dot{x}x+\dot{y}y)}{(x^2+y^2)^{1/2}} - \frac{1}{\gamma(x^2+y^2)^{1/2}} \quad -(16a)$$

so

$$\frac{\dot{x}}{x} = \frac{\dot{y}}{y} \quad -(17)$$

As shown in Note 377(7) this equation is satisfied
y the ellipse:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1. \quad -(18)$$

ifferentiating:

$$b^2 x\dot{x} + a^2 y\dot{y} = 0 \quad -(19)$$

Multiply by $\dot{x}$:

$$a^2 y\dot{y}\dot{x} = -b^2 x\dot{x}^2 \quad -(20)$$

so

$$\dot{x}y = -\frac{b^2}{a^2}\frac{x\dot{x}^2}{\dot{y}}. \quad -(21)$$

ultiply eq. (19) by $x$:

$$x\dot{y} = -\frac{b^2}{a^2}\frac{x^2\dot{x}}{y} \quad -(22)$$

) If $$x\dot{y} = y\dot{x} \quad -(23)$$

[...]ter from eqs. (21) and (22):

$$\frac{x\dot{x}^2}{\dot{y}} = \frac{x^2\dot{x}}{y} \quad -(24)$$

i.e.

$$xy\dot{x}^2 = \dot{y}\dot{x}x^2 \quad -(25)$$

or

$$(\dot{x}y)(\dot{x}x) = \dot{y}x(\dot{x}x) \quad -(26)$$

[...]elf consistently, Q.E.D.

This means that the vector potential (10) and [...]calar spin connection (16) produce a Newtonian ellipse [...]elf consistently in this first approximation. A more general [...]olution must now be sought from:

$$\underline{g} = \ddot{\underline{r}} = \frac{mG}{\gamma r^3 c^2}\,\dot{\underline{r}}\,(\dot{\underline{r}}\cdot\underline{r}) - \frac{mG}{\gamma}\frac{\underline{r}}{r^3} \quad -(27)$$

$$= -\,\omega_0\,\underline{Q}$$

The approximate solutions (10) and (16) obey antisymmetry.