384(5): Spin Connections for Retrograde and Forward Precession.

These are calculated from:

$$\underline{g} = -\omega_0 \underline{c} \quad -(1)$$

and

$$\omega_0 = -\frac{c}{r}. \quad -(2)$$

For retrograde precession:

$$\underline{g} = -\frac{mG}{\gamma^3}\frac{\underline{r}}{r^3} \quad -(3)$$

so:

$$-\frac{mG}{\gamma^3}\frac{\underline{r}}{r^3} = \frac{c}{r}\underline{c} \quad -(4)$$

so

$$\underline{c} = -\frac{mG}{\gamma^3 c}\frac{\underline{r}}{r^2} \quad -(5)$$

where

$$\gamma = \left(1 - \frac{\dot{x}^2 + \dot{y}^2}{c^2}\right)^{-1/2} \quad -(6)$$

The spin connection components are found from:

$$\frac{\partial Q_Y}{\partial X} = \omega_X Q_Y \quad -(7)$$

and

$$\frac{\partial Q_X}{\partial Y} = \omega_Y Q_X \quad -(8)$$

It follows that

$$\boxed{\underline{\omega} = -2\frac{\underline{r}}{r^2}} \quad -(9)$$

for retrograde precession, and the spin connection is the same as in the Newtonian result.

For forward precession:

$$\underline{g} = \frac{mG}{\gamma r^3}\left(\frac{\underline{\dot{r}}(\underline{\dot{r}}\cdot\underline{r})}{c^2} - \underline{r}\right) = \frac{c}{r}\underline{Q} \quad -(10)$$

or:

$$\underline{Q} = \frac{mG}{\gamma c r^2}\left(\frac{\underline{\dot{r}}(\underline{r}\cdot\underline{r})}{c^2} - \underline{r}\right). \quad -(11)$$

It follows that:

$$Q_X = \frac{mG}{\gamma c(X^2+Y^2)^{1/2}}\left(\frac{\dot{X}\dot{Y}Y + X\dot{X}^2}{c^2} - X\right) -(12)$$

and

$$Q_Y = \frac{mG}{\gamma c(X^2+Y^2)^{1/2}}\left(\frac{\dot{Y}\dot{X}X + Y\dot{Y}^2}{c^2} - Y\right) -(13)$$

Since γ has no dependence on X and Y:

$$\frac{\partial Q_X}{\partial Y} = \frac{mG}{\gamma c}\frac{\partial}{\partial Y}\left(\frac{\dot{X}\dot{Y}Y + X\dot{X}^2}{c^2(X^2+Y^2)^{1/2}} - X\right) -(14)$$

$$\frac{\partial Q_Y}{\partial X} = \frac{mG}{\gamma c}\frac{\partial}{\partial X}\left(\frac{Y\dot{X}X + Y\dot{Y}^2}{c^2(X^2+Y^2)^{1/2}} - Y\right), -(15)$$

$$\frac{\partial Q_X}{\partial Y} = \frac{mG}{\gamma c^3(X^2+Y^2)^{1/2}}\left(\dot{X}\dot{Y} - \frac{Y\dot{X}(Y\dot{Y}+X\dot{X})}{X^2+Y^2}\right)$$

$$= \omega_X Q_Y$$

$$= \omega_x \frac{mG}{r c^3 (x^2+y^2)^{1/2}} \left(\dot{y}(x\dot{x}+y\dot{y}) - c^2 y \right) \quad -(16)$$

so

$$\omega_x = \frac{\dot{x}\dot{y} - y\dot{x}\dfrac{(y\dot{y}+x\dot{x})}{x^2+y^2}}{\dot{y}(x\dot{x}+y\dot{y}) - c^2 y} \quad -(17)$$

Similarly: $-(18)$

$$\omega_y = \frac{\dot{y}\dot{x} - x\dot{x}(y\dot{y}+x\dot{x})/(x^2+y^2)}{\dot{x}(y\dot{y}+x\dot{x}) - c^2 x}$$

and

$$\underline{\omega} = \omega_x \underline{i} + \omega_y \underline{j} \quad -(19)$$

$$= \left(\frac{\dfrac{\dot{x}\dot{y}}{y\dot{y}+x\dot{x}} - \dfrac{y\dot{x}}{x^2+y^2}}{\dot{y} - c^2\left(\dfrac{y}{x\dot{x}+y\dot{y}}\right)} \right) \underline{i} + \left(\frac{\dfrac{\dot{y}\dot{x}}{y\dot{y}+x\dot{x}} - \dfrac{x\dot{x}}{x^2+y^2}}{\dot{x} - c^2\dfrac{x}{x\dot{x}+y\dot{y}}} \right) \underline{j}$$

These calculations can be checked by computer algebra and graphed and analyzed. They show that both forward and retrograde precessions obey antisymmetry under the assumption that there is no gravitomagnetic field and that $\omega_0 = -c/r$.