84(4): Complete Solution of the ECE2 Field Equations for Planar Orbits

Consider for illustration the Newtonian orbit:

$$\underline{g} = -\underline{\nabla}\,\Phi + \underline{\omega}\,\Phi = -\frac{\partial \underline{Q}}{\partial t} - \omega_0 \underline{Q} \quad -(1)$$

by antisymmetry. Assume that:

$$\frac{\partial \underline{Q}}{\partial t} = \underline{0} \quad -(2)$$

for the gravitostatic $\underline{g}$. Then:

$$\underline{g} = -\underline{\nabla}\,\Phi + \underline{\omega}\,\Phi = -\omega_0 \underline{Q} \quad -(3)$$

which has the solution of the previous notes:

$$\omega_0 = -\frac{c}{r} \quad -(4)$$

$$\underline{Q} = -\frac{mG}{c}\frac{\underline{r}}{r^2} \quad -(5)$$

and

$$\underline{g} = -mG\frac{\underline{r}}{r^3} \quad -(6)$$

Now assume that ω_0 is universal and is the same for all types of planar orbit. Each orbit is therefore characterised by $\underline{Q}$, which is different for each orbit.

For a planar orbit:

$$\underline{r} = X\underline{i} + Y\underline{j} \quad -(7)$$

so:

2)

$$Q_x = -\frac{mG}{c}\frac{X}{X^2+Y^2} \quad -(8)$$

$$Q_Y = -\frac{mG}{c}\frac{Y}{X^2+Y^2} \quad -(9)$$

By antisymmetry:

$$\left(\frac{\partial}{\partial Y}-\omega_Y\right)Q_Z = -\left(\frac{\partial}{\partial Z}-\omega_Z\right)Q_Y \quad -(10)$$

$$\left(\frac{\partial}{\partial Z}-\omega_Z\right)Q_X = -\left(\frac{\partial}{\partial X}-\omega_X\right)Q_Z \quad -(11)$$

$$\left(\frac{\partial}{\partial X}-\omega_X\right)Q_Y = -\left(\frac{\partial}{\partial Y}-\omega_Y\right)Q_X \quad -(12)$$

For a planar orbit:

$$\left(\frac{\partial}{\partial X}-\omega_X\right)Q_Y = -\left(\frac{\partial}{\partial Y}-\omega_Y\right)Q_X \quad -(13)$$

and Eqs. (10) and (11) each reduce to zero on both sides.

The antisymmetry law for planar orbits is therefore:

$$\boxed{\frac{\partial Q_Y}{\partial X}+\frac{\partial Q_X}{\partial Y} = \omega_X Q_Y + \omega_Y Q_X} \quad -(14)$$

In the absence of a gravitomagnetic field:

$$\underline{\Omega} = \underline{\nabla}\times\underline{Q} - \underline{\omega}\times\underline{Q} = \underline{0} \quad -(15)$$

This means that:

$$\frac{\partial Q_Z}{\partial Y} - \frac{\partial Q_Y}{\partial Z} = \omega_Y Q_Z - \omega_Z Q_Y \quad -(16)$$

$$\frac{\partial Q_X}{\partial Z} - \frac{\partial Q_Z}{\partial X} = \omega_Z Q_X - \omega_X Q_Z \quad -(17)$$

$$\frac{\partial Q_Y}{\partial X} - \frac{\partial Q_X}{\partial Y} = \omega_X Q_Y - \omega_Y Q_X \quad -(18)$$

For a planar orbit the left side of eqs. (16) and (17) reduce to zero, so in the absence of a gravitomagnetic field:

$$\boxed{\frac{\partial Q_Y}{\partial X} - \frac{\partial Q_X}{\partial Y} = \omega_X Q_Y - \omega_Y Q_X} \quad -(19)$$

From eqs. (14) and (19):

$$\frac{\partial Q_Y}{\partial X} = \omega_X Q_Y \quad -(20)$$

$$\frac{\partial Q_X}{\partial Y} = \omega_Y Q_X \quad -(21)$$

From eqs. (8) and (9):

$$\frac{\partial Q_Y}{\partial X} = \frac{mG}{c} \frac{2XY}{(X^2+Y^2)^2} \quad -(22)$$

$$\frac{\partial Q_X}{\partial Y} = \frac{mG}{c} \frac{2YX}{(X^2+Y^2)^2} \quad -(23)$$

so

$$\omega_X = -\frac{2X}{(X^2+Y^2)} \quad -(24)$$

$$\omega_Y = -\frac{2Y}{(X^2+Y^2)} \quad -(25)$$

4) and

$$\boxed{\underline{\omega} = -\frac{2\underline{r}}{r^2}} \quad -(26)$$

It is again assumed that this is an intrinsic property of each planar orbit. Therefore the spin connection four vector is

$$\boxed{\omega^\mu = \left(\frac{\omega_0}{c}, \underline{\omega}\right) = -\left(\frac{1}{r}, \frac{2\underline{r}}{r^2}\right)} \quad -(27)$$

for Newtonian orbit.

Finally the scalar potential is calculated from

$$\begin{aligned} \underline{g} &= -\underline{\nabla}\underline{\Phi} + \underline{\omega}\,\underline{\Phi} \\ &= -\underline{\nabla}\underline{\Phi} - 2\underline{\Phi}\frac{\underline{r}}{r^2} \\ &= -MG\frac{\underline{r}}{r^3} \end{aligned} \quad -(28)$$

It follows that:

$$\underline{\Phi} = \frac{MG}{r} \quad -(29)$$

This solution obeys the antisymmetry laws (1) and (14).

Note carefully that the sign of $\underline{\Phi}$ is opposite to that of the standard model, in

) which there is no spin connection for Newtonian gravitation. In ECE2 gravitation the spin connection for Newtonian planar orbits is the ______ eq. (27).

In this solution:

$$\underline{\nabla} \times (\omega_0 \underline{Q}) = \underline{0} \quad -(30)$$

and

$$\underline{\nabla} \cdot (\omega_0 \underline{Q}) = -4\pi G \rho_m \quad -(31)$$

as in previous notes.

<u>For retrograde precession:</u>

$$\underline{g} = -\frac{mG}{\gamma^3} \frac{\underline{r}}{r^3} = -\omega_0 \underline{Q} \quad -(32)$$

$$= \frac{c}{r} \underline{Q}$$

so

$$\boxed{\underline{Q} = -\frac{mG}{\gamma^3 c} \frac{\underline{r}}{r^2}} \quad -(33)$$

and this obeys eqs. (14) and (19) provided that the spin connection for retrograde precession is defined by eqs. (20) and (21).

So retrograde precession obeys antisymmetry.

<u>For forward precession:</u>

$$\underline{g} = \frac{mG}{\gamma r^3} \left(\frac{\underline{\dot{r}} (\underline{\dot{r}} \cdot \underline{r})}{c^2} - \underline{r} \right) = -\omega_0 \underline{Q}$$

$$= \frac{c}{r} \underline{Q} \quad -(34)$$

So forward precession in the absence of a gravitomagnetic field obeys antisymmetry provided that the spin connection is defined by eqs. (20) and (21).

So forward precession obeys antisymmetry.

It has been assumed that the scalar spin connection:

$$\omega_0 = -\frac{c}{r} \quad -(35)$$

is the same for all three types of orbit. Its modulus is a fundamental angular frequency of spacetime:

$$\Omega_0 = 2\pi \left| \omega_0 \right| = 2\pi \frac{c}{r} \quad -(36)$$

This is the angular frequency of the vacuum particle:

$$E = \hbar \Omega_0 = \gamma m c^2 \quad -(37)$$