384(2): Check of Concepts in Note 384(1) and UFT381

The relevant equations are:

$$\frac{\partial}{\partial t}\left(\underline{\nabla}\times\underline{Q}\right)+\underline{\nabla}\times\left(\omega_0\,\underline{Q}\right)=\underline{0}\quad-(1)$$

and

$$\frac{\partial \underline{Q}}{\partial t}=\underline{0}\quad-(2)$$

so it follows that

$$\underline{\nabla}\times\left(\omega_0\,\underline{Q}\right)=\underline{0}\quad-(3)$$

The solution of Eq. (3) used in Note 384(1) and UFT381 is:

$$\underline{Q}=\frac{mG}{\omega_0}\frac{\underline{r}}{r^3}\quad-(4)$$

and

$$\omega_0=-\frac{c}{r}\quad-(5)$$

so

$$\omega_0\,\underline{Q}=mG\frac{\underline{r}}{r^3}\quad-(6)$$

and

$$\underline{\nabla}\times\omega_0\,\underline{Q}=mG\,\underline{\nabla}\times\frac{\underline{r}}{r^3}=\underline{0}\quad-(7)$$

Q.E.D.

The vector potential is:

$$\boxed{\underline{Q}=-\frac{mG}{c}\frac{\underline{r}}{r^2}}\quad-(8)$$

and the S.I. units are:

$$mG=c^2r=m^3s^{-2},\quad Q=ms^{-1}\quad-(9)$$

The Coulomb law gives:

$$\underline{\nabla}\cdot\underline{g}=\underline{\nabla}\cdot\left(-\frac{\partial\underline{Q}}{\partial t}-\omega_0\,\underline{Q}\right)=4\pi G\rho_m$$

so

$$\underline{\nabla}\cdot\left(\omega_0\,\underline{Q}\right)=-4\pi G\rho_m\quad-(10)$$

) In general:

$$\underline{\nabla} \times (\omega_0 \underline{Q}) = \underline{0} \qquad -(11)$$

$$\underline{\nabla} \cdot (\omega_0 \underline{Q}) = -4\pi G \rho_m \qquad -(12)$$

and these must be solved simultaneously for $\omega_0$ and $\underline{Q}$.

Using eq. (6):

$$m G \, \underline{\nabla} \cdot \frac{\underline{r}}{r^3} = -4\pi G \rho_m \qquad -(13)$$

so

$$\rho_m = -\frac{m}{4\pi} \left( \underline{\nabla} \cdot \frac{\underline{r}}{r^3} \right), \qquad -(14)$$

where

$$\underline{\nabla} \cdot \frac{\underline{r}}{r^3} = \frac{\partial}{\partial x} \left( \frac{x}{(x^2+y^2)^{3/2}} \right) + \frac{\partial}{\partial y} \left( \frac{y}{(x^2+y^2)^{3/2}} \right)$$

$$= \frac{2(x^2+y^2)^{3/2} - 3x^2(x^2+y^2)^{1/2} - 3y^2(x^2+y^2)^{1/2}}{(x^2+y^2)^3}$$

$$= -\frac{4}{(x^2+y^2)^{3/2}} \qquad -(15)$$

so

$$\boxed{\rho_m = \frac{m}{\pi (x^2+y^2)^{3/2}}} \qquad -(16)$$

Therefore this solution is self consistent, QED