84(1): Antisymmetry Laws for Precessing Orbits

For precessing orbits:

$$\gamma^3 \underline{g} = -\underline{\nabla}\Phi + \underline{\omega}\Phi = -\frac{\partial \underline{Q}}{\partial t} - \omega_0 \underline{Q} \quad -(1)$$

As in the case of the static gravitational field, assume:

$$-\underline{\nabla}\Phi = \underline{\omega}\Phi \quad -(2)$$

$$\frac{\partial \underline{Q}}{\partial t} = \underline{0}. \quad -(3)$$

then:

$$\gamma^3 \underline{g} = -2\underline{\nabla}\Phi = 2\underline{\omega}\Phi = -\omega_0 \underline{Q} \quad -(4)$$

A solution is:

$$\Phi = -\frac{mG}{2r}, \quad \underline{\omega} = \frac{\underline{r}}{r^2}. \quad -(5)$$

It follows that:

$$Q_X = \frac{mG}{\omega_0}\frac{X}{(X^2+Y^2)^{3/2}} \quad -(6)$$

$$Q_Y = \frac{mG}{\omega_0}\frac{Y}{(X^2+Y^2)^{3/2}} \quad -(7)$$

and

$$\underline{g} = -\frac{\omega_0}{\gamma^3}\underline{Q} \quad -(8)$$

From eq. (4) it follows that

$$\underline{\omega} \times \underline{Q} = \underline{0} \quad -(9)$$

From eqs. (6) and (7):

$$\frac{\partial Q_X}{\partial Y} = \frac{\partial Q_Y}{\partial X} \quad -(10)$$

Therefore:

$$\frac{\partial a_Y}{\partial X} - \frac{\partial a_X}{\partial Y} = \omega_X a_Y - \omega_Y a_X = 0 \quad -(11)$$

Q.E.D., for all ω_0, and antisymmetry is obeyed.

As in Note 381(5):

$$\underline{\nabla} \times \underline{g} = \underline{0} \quad -(12)$$

$$\underline{\nabla} \cdot \underline{g} = 4\pi G \rho_m \quad -(13)$$

$$\frac{\partial \underline{g}}{\partial t} = \underline{0} \quad -(14)$$

$$\underline{\Omega} = \underline{\nabla} \times \underline{Q} \qquad \underline{\omega} \times \underline{Q} = \underline{0} \quad -(15)$$

From eqs. (12) and (15):

$$\frac{\partial}{\partial t}\left(\underline{\nabla} \times \underline{Q}\right) + \underline{\nabla} \times \left(\omega_0 \underline{Q}\right) = \underline{0} \quad -(16)$$

Using eq. (3):

$$\underline{\nabla} \times \left(\omega_0 \underline{Q}\right) = \underline{0} \quad -(17)$$

i.e. $$\omega_0 \underline{\nabla} \times \underline{Q} + \underline{Q} \times \underline{\nabla}\, \omega_0 = \underline{0}. \quad -(18)$$

A solution of eqs. (17) and (18) is:

$$\omega_0 = -\frac{c}{r}, \quad -(19)$$

so

$$a_X = -\frac{mG}{c}\frac{X}{(X^2+Y^2)} \quad -(20)$$

$$a_Y = -\frac{mG}{c}\frac{Y}{X^2+Y^2} \quad -(21)$$

The basic equation for retrograde precession is as follows:

3)

$$\frac{d\underline{p}}{dt} = \frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \underline{\dot{r}}} = \gamma^3 m \underline{\ddot{r}} \quad -(22)$$

where:

$$\underline{p} = \gamma m \underline{v} = \gamma m \underline{\dot{r}} \quad -(23)$$

and

$$\mathcal{L} = -mc^2 \left(1 - \frac{v^2}{c^2}\right)^{1/2} + \frac{mMG}{r} \quad -(24)$$

Therefore the phenomenon of retrograde precession obeys ECE2 antisymmetry, Q.E.D.