351(2): Units Analysis and Conversion Factors

As in note 351(1), the minimal prescription can be used to show that:

$$\underline{v} = \frac{e}{m}\underline{A} \quad -(1)$$

For a continuum:

$$\underline{v} = \frac{\rho}{\rho_m}\underline{A} \quad -(2)$$

where ρ is current density and ρ_m is mass density. The basic S.I. units are, for electrodynamics:

$$E = \text{volt}\, m^{-1} = J C^{-1} m^{-1}$$

$$A = J s C^{-1} m^{-1}$$

$$\phi_W = J C^{-1}$$

$$B = J s C^{-1} m^{-2}$$

$$= C m^{-3}$$

$$\rho_J = C m^{-2} s^{-1}$$

$$\epsilon_0 = J^{-1} C^2 m^{-1}$$

$$\mu_0 = J s^2 C^{-2} m^{-1}$$

The Kambe electric field is:

$$\underline{E}_F = -\frac{\partial \underline{v}}{\partial t} - \underline{\nabla} h = (\underline{v}\cdot\underline{\nabla})\underline{v} \quad -(3)$$

where

$$\underline{\nabla} h = \frac{1}{\rho_m}\underline{\nabla} \underline{p} \quad -(4)$$

where $\underline{p}$ is the pressure.

The ECE2 electric field strength is volt m^{-1} is

$$\underline{E} = -\underline{\nabla}\phi_W - \frac{\partial \underline{W}}{\partial t} \quad -(5)$$

From eqns (3) and (5):

2)

$$\boxed{\underline{W} = \frac{\rho_m}{\rho}\,\underline{v}} \quad -(6)$$

It follows that $\boxed{\phi_W = \frac{\rho_m}{\rho}\,h}$ $-(7)$

Units check $\quad \phi_W = JC^{-1} = \frac{kgm}{C}\,J\,kgm^{-1}$ ✓✓

because h is the enthalpy per unit mass.

From eq. (6): $\boxed{\underline{B} = \frac{\rho_m}{\rho}\,\underline{H} = \frac{\rho_m}{\rho}\,\underline{w}}$ $-(8)$

here

$$\underline{w} = \underline{\nabla} \times \underline{v} \quad -(9)$$

is the vorticity.

It also follows that:

$$\boxed{\underline{E} = \frac{\rho_m}{\rho}\,\underline{E}_F} \quad -(10)$$

and

$$\underline{\nabla} \cdot \underline{E} = \frac{\rho}{\epsilon_0} = \frac{\rho_m}{\rho}\,q \quad -(11)$$

so

$$\boxed{\rho = \epsilon_0 \frac{\rho_m}{\rho}\,q} \quad -(12)$$

The inhomogeneous field equation of Kambe is:

3)

$$a_0^2 \, \underline{\nabla} \times \underline{H}_F - \frac{\partial \underline{E}_F}{\partial t} = \underline{J}_F \quad -(13)$$

and the inhomogeneous ECE2 field equation is:

$$\underline{\nabla} \times \underline{B} - \frac{1}{c^2} \frac{\partial \underline{E}}{\partial t} = \mu_0 \underline{J} \quad -(14)$$

Using eqs. (8) and (10), eq. (13) becomes:

$$\frac{\rho}{\rho_m} \left(\underline{\nabla} \times \underline{B} - \frac{1}{a_0^2} \frac{\partial \underline{E}}{\partial t} \right) = \frac{1}{a_0^2} \underline{J}_F \quad -(15)$$

where a_0 is the speed of sound.

Define the refractive index as:

$$\boxed{n^2 = \frac{c^2}{a_0^2}} \quad -(16)$$

Then eq. (15) is:

$$\underline{\nabla} \times \underline{B} - \frac{1}{a_0^2} \frac{\partial \underline{E}}{\partial t} = \left(\frac{\rho_m}{\rho a_0^2} \right) \underline{J}_F \quad -(17)$$

$$= \underline{\nabla} \times \underline{B} - \frac{n^2}{c^2} \frac{\partial \underline{E}}{\partial t} = \mu \underline{J}$$

where μ is the permeability of the medium - the fluid.

so

$$\boxed{\underline{J} = \frac{1}{\mu} \frac{\rho_m}{\rho} \underline{J}_F} \quad -(18)$$

Therefore to convert any hydrodynamic equation into an equation of electromagnetism use:

$$\underline{V} = \frac{\rho}{\rho_m} \underline{W} \quad -(19)$$

$$h = \frac{\rho}{\rho_m} \phi_W \quad -(20)$$

$$\underline{H}_F = \frac{\rho}{\rho_m} \underline{B} \quad -(21)$$

$$\underline{E}_F = \frac{\rho}{\rho_m} \underline{E} \quad -(22)$$

$$q = \frac{1}{\epsilon_0} \frac{\rho}{\rho_m} \rho \quad -(23)$$

$$\underline{J}_F = \mu \frac{\rho}{\rho_m} \underline{J} \quad -(24)$$

where the refractive index is:

$$n = \left(\frac{\rho}{\rho_m} \right)^{1/2} \quad -(25)$$

So the Kambe equations translate to:

$$\underline{\nabla} \cdot \underline{B} = 0 \quad -(26)$$

$$\underline{\nabla} \cdot \underline{E} = \rho / \epsilon_0 \quad -(27)$$

$$\underline{\nabla} \times \underline{E} + \frac{\partial \underline{B}}{\partial t} = \underline{0} \quad -(28)$$

$$\underline{\nabla} \times \underline{B} - \frac{n^2}{c^2} \frac{\partial \underline{E}}{\partial t} = \mu \underline{J} \quad -(29)$$

where

$$n = c / a_0 \quad -(30)$$

The Euler equation of ______ hydrodynamics is

$$\frac{D\underline{v}}{Dt} = \underline{\nabla} h \quad -(31)$$

5) where

$$\underline{\nabla} h = \frac{1}{\rho_m} \underline{\nabla} p \quad -(32)$$

and

$$\frac{D\underline{v}}{Dt} = \frac{\partial \underline{v}}{\partial t} + (\underline{v} \cdot \underline{\nabla}) \underline{v} \quad -(33)$$

is the convective derivative. So eq. (31) becomes the following equation of electrodynamics:

$$\boxed{\frac{D\underline{\overline{W}}}{Dt} = \frac{\partial \underline{\overline{W}}}{\partial t} + (\underline{\overline{W}} \cdot \underline{\nabla}) \underline{\overline{W}} = \underline{\nabla} \phi_W} \quad -(34)$$

which is a new relation between ϕ_W and $\underline{\overline{W}}$ in spacetime.

In order to describe electrodynamical or gravitational turbulence use the relevant equations of hydrodynamics and translate into electrodynamics or gravitational physics.