316(5): New Vector Equation from the Space Like Part of the Cartan Identity

As demonstrated in UFT 254 and UFT 255 the space like part of the Cartan identity is:

$$\underline{\nabla} \cdot \underset{\sim}{T}^a + \underline{\omega}^a{}_b \cdot \underset{\sim}{T}^b = \underline{q}^b \cdot \underline{R}^a{}_b(\text{spin}) \quad -(1)$$

where:

$$\underline{T}^b = \underline{\nabla} \times \underline{q}^b - \underline{\omega}^b{}_c \times \underline{q}^c \quad -(2)$$

and

$$\underline{R}^a{}_b(\text{spin}) = \underline{\nabla} \times \underline{\omega}^a{}_b - \underline{\omega}^a{}_c \times \underline{\omega}^c{}_b \quad -(3)$$

Therefore:

$$\underline{\nabla} \cdot \left(\underline{\nabla} \times \underline{q}^a - \underline{\omega}^a{}_b \times \underline{q}^b\right) + \underline{\omega}^a{}_b \cdot \left(\underline{\nabla} \times \underline{q}^b - \underline{\omega}^b{}_c \times \underline{q}^c\right)$$
$$= \underline{q}^b \cdot \left(\underline{\nabla} \times \underline{\omega}^a{}_b - \underline{\omega}^a{}_c \times \underline{\omega}^c{}_b\right) \quad -(4)$$

Now use:

$$\underline{\nabla} \cdot \underline{\nabla} \times \underline{q}^a = 0, \quad -(5)$$

with:

$$\underline{\nabla} \cdot \underline{q}^b \times \underline{\omega}^a{}_b = \underline{\omega}^a{}_b \cdot \underline{\nabla} \times \underline{q}^b - \underline{q}^b \cdot \underline{\nabla} \times \underline{\omega}^a{}_b \quad -(6)$$

to find that:

$$-\underline{\nabla} \cdot \underline{\omega}^a{}_b \times \underline{q}^b + \underline{\nabla} \cdot \underline{q}^b \times \underline{\omega}^a{}_b$$
$$= \underline{\omega}^a{}_b \cdot \underline{\omega}^b{}_c \times \underline{q}^c - \underline{q}^b \cdot \underline{\omega}^a{}_c \times \underline{\omega}^c{}_b$$
$$= 2\,\underline{\nabla} \cdot \underline{q}^b \times \underline{\omega}^a{}_b \quad -(7)$$

d) Finally use:

$$\underline{\omega}^a{}_b \cdot \underline{\omega}^b{}_c \times \underline{q}^c = \underline{q}^b \cdot (\underline{\omega}^a{}_c \times \underline{\omega}^c{}_b) \quad -(8)$$

to find that eq. (1) becomes:

$$\boxed{\underline{\nabla} \cdot \underline{q}^b \times \underline{\omega}^a{}_b = 0} \quad -(9)$$

This is the result found in UFT 254 and UFT255, Q.E.D, and is the space-like part of the Cartan identity.

Now use:

$$\underline{W}^a{}_b = W^{(0)} \underline{\omega}^a{}_b \text{ tesla metres} \quad -(10)$$

$$\underline{A}^b = A^{(0)} \underline{q}^b \text{ tesla metres} \quad -(11)$$

to find that:

$$\boxed{\underline{\nabla} \cdot \underline{A}^b \times \underline{W}^a{}_b = 0} \quad -(12)$$

A possible solution of eq. (12) is:

$$\underline{A}^b \times \underline{W}^a{}_b = \underline{0} \quad -(13)$$

meaning that $\underline{W}^a{}_b$ is parallel to $\underline{A}^b$.

The tangent indices can be removed using:

$$\underline{B} = -e_a \underline{B}^a \quad -(14)$$

as in Note 316(4).

3)

It follows that:

$$\boxed{\underline{\nabla} \cdot \underline{A} \times \underline{W} = 0} \quad -(16)$$

and:

$$\boxed{\underline{\nabla} \cdot \underline{q} \times \underline{\omega} = 0} \quad -(17)$$

Possible solutions are:

$$\underline{A} \times \underline{W} = \underline{0} \quad -(18)$$

and

$$\underline{q} \times \underline{\omega} = \underline{0} \quad -(19)$$

i.e $\underline{A}$ is parallel to $\underline{W}$ and $\underline{q}$ is parallel to $\underline{\omega}$.

Finally assume that $\underline{\omega}^a{}_b$ is dual to $\underline{q}^c$ in the tangent space:

$$r^{(0)} \underline{\omega}^a{}_b = \epsilon^a{}_{bc} \underline{q}^c \quad -(20)$$

and

$$\underline{q}_c = r^{(0)} \epsilon^b{}_{ac} \underline{\omega}^a{}_b \quad -(21)$$

Then eq. (9) becomes:

$$\underline{\nabla} \cdot \underline{q}^b \times \underline{q}^c = 0 \quad -(21)$$

Removing the indices gives:

$$\underline{\nabla} \cdot \underline{q} \times \underline{q} = 0 \quad -(22)$$

<u>Q.E.D</u>

Eq. (20) implies that $\underline{W}^a{}_b$ is dual to $\underline{A}^c$:

$$\underline{W}^a{}_b = \epsilon^a{}_{bc}\,\underline{A}^c \quad -(23)$$

So

$$\underline{\nabla}\cdot\underline{A}^b\times\underline{A}^c = 0 \quad -(24)$$

or

$$\underline{\nabla}\cdot\underline{A}\times\underline{A} = 0 \quad -(25)$$

In this case $\underline{W}^a{}_b$ is parallel and equal to $\underline{A}^c$.

If eq. (23) is assumed the torsion and curvature based ECE <u>theories</u> become equivalent.