238(9): Relativistic Dynamics of a Whirlpool Galaxy

In note 238(8) it was found that:

$$r = \left(1 - \left(\frac{L_0}{m r_0 c}\right)^2 \left(1 + \left(\frac{r_0}{r}\right)^2\right)\right)^{1/2} \frac{L_0}{m r_0} t \quad -(1)$$

This can be solved for r^2 in terms of t as follows. First note that:

$$r^2 = \left(1 - \left(\frac{L_0}{m r_0 c}\right)^2 \left(\frac{r^2 + r_0^2}{r^2}\right)\right) \left(\frac{L_0}{m r_0}\right)^2 t^2 \quad -(2)$$

i.e.

$$r^4 = \left(r^2 - \left(\frac{L_0}{m r_0 c}\right)^2 \left(r^2 + r_0^2\right)\right) \left(\frac{L_0}{m r_0}\right)^2 t^2$$

$$= r^2 \left(1 - \left(\frac{L_0}{m r_0 c}\right)^2\right) t^2 \left(\frac{L_0}{m r_0}\right)^2 - \left(\frac{L_0}{m r_0 c}\right)^2 \left(\frac{L_0}{m r_0}\right)^2 r_0^2 t^2 \quad -(3)$$

i.e.

$$r^4 = A r^2 t^2 - B t^2 \quad -(4)$$

where

$$A = \left(1 - \left(\frac{L_0}{m r_0 c}\right)^2\right) \left(\frac{L_0}{m r_0}\right)^2, \quad -(5)$$

$$B = \left(\frac{L_0}{m r_0 c}\right)^2 \left(\frac{L_0}{m r_0}\right)^2 r_0^2 \quad -(6)$$

So from eq. (4):

2)

$$r^2 = \frac{1}{2}\left(At^2 \pm \left(A^2t^4 - 4Bt^2\right)^{1/2}\right) \quad -(7)$$

i.e.

$$r^2 = \frac{1}{2}\left(At^2 \pm t\left(A^2t^2 - 4B\right)^{1/2}\right) \quad -(8)$$

Finally:

$$\boxed{r = \frac{1}{\sqrt{2}}\left(At^2 \pm t\left(A^2t^2 - 4B\right)^{1/2}\right)^{1/2}} \quad -(9)$$

This eq. gives the dependence of r on t. In the non-relativistic limit it reduces to:

$$r = \left(\frac{L_0}{m r_0}\right)t \quad -(10)$$

if the positive root is used, so:

$$\boxed{r = \frac{t}{\sqrt{2}}\left(At + \left(A^2t^2 - 4B\right)^{1/2}\right)^{1/2}} \quad -(11)$$

In the non-relativistic limit:

$$A \rightarrow \left(\frac{1}{m r_0}\right)^2, \quad B \rightarrow 0. \quad -(12)$$