238(6): Summary and Check of Results.

The relativistic force is:

$$\underline{F} = m\underline{a} = m\frac{d}{d\tau}\left(\frac{d\underline{r}}{d\tau}\right) \quad -(1)$$

where τ is the proper time. Eq. (1) is the space part of the Minkowski four-force. In terms of the time t in the laboratory frame:

$$\underline{F} = \gamma m\left(\frac{d\gamma}{dt}\frac{d\underline{r}}{dt} + \gamma\frac{d}{dt}\left(\frac{d\underline{r}}{dt}\right)\right) \quad -(2)$$

in which the Lorentz factor is:

$$\gamma = \left(1 - \frac{v^2}{c^2}\right)^{-1/2} \quad -(3)$$

and:

$$ds^2 = c^2 d\tau^2 = c^2 dt^2 - d\underline{r}\cdot d\underline{r}, \quad -(4)$$

$$d\underline{r}\cdot d\underline{r} = dr^2 + r^2 d\theta^2 = v^2 dt^2 \quad -(5)$$

so

$$v = \left(\left(\frac{dr}{dt}\right)^2 + r^2\left(\frac{d\theta}{dt}\right)^2\right)^{1/2} \quad -(6)$$

In eq (2):

$$\frac{d\gamma}{dt} = \gamma^3\frac{v}{c^2}\frac{dv}{dt} \quad -(7)$$

so

$$\underline{F} = m\left(\gamma^4\frac{v}{c^2}\frac{dv}{dt}\frac{d\underline{r}}{dt} + \gamma^2\frac{d}{dt}\frac{d\underline{r}}{dt}\right) \quad -(8)$$

in which:

2)

in which:

$$\frac{d\underline{r}}{dt} = \frac{dr}{dt}\underline{e}_r + \underline{\omega} \times \underline{r} \quad -(9)$$

$$\frac{d^2\underline{r}}{dt^2} = \frac{d^2 r}{dt^2}\underline{e}_r + \underline{\omega} \times (\underline{\omega} \times \underline{r}) + \frac{d\underline{\omega}}{dr} \times \underline{r} + 2\underline{\omega} \times \frac{dr}{dt}\underline{e}_r$$

$-(10)$

For planar orbits:

$$\underline{F} = m\left(\left(\gamma^4 \frac{d^2 r}{dt^2} - \frac{L^2}{m^2 r^3}\right)\underline{e}_r + \frac{\gamma^4}{c^2}\frac{dr}{dt}\frac{d^2 r}{dt^2}\,\omega r\,\underline{e}_\theta\right)$$

$$= -\frac{L^2}{mr^2}\left(\gamma^2 \frac{d^2}{d\theta^2}\left(\frac{1}{r}\right) + \frac{1}{r}\right)\underline{e}_r \quad -(11).$$

$$+ \frac{L^4}{c^2 m^3 r^3}\frac{d}{d\theta}\left(\frac{1}{r}\right)\frac{d^2}{d\theta^2}\left(\frac{1}{r}\right)\underline{e}_\theta$$

in which

$$L = mr^2 \frac{d\theta}{d\tau} = \underline{\text{constant}} \text{ of } \underline{\text{motion}}$$

$-(12)$
