

238(10) : Time Evolution in Cartesian Coordinates.

In plane polar coordinates :

$$\underline{r}(t) = r(t) \underline{e}_r \quad - (1)$$

For purpess of animation and visualization it is convenient to transform to Cartesian coordinates :

$$\underline{e}_r = \cos \theta \underline{i} + \sin \theta \underline{j} \quad - (2)$$

$$\underline{e}_\theta = -\sin \theta \underline{i} + \cos \theta \underline{j} \quad - (3)$$

so
$$\underline{r}(t) = r(t) (\cos \theta \underline{i} + \sin \theta \underline{j}) \quad - (4)$$

For a hyperbolic spiral :

$$r = -r_0 / \theta, \quad - (5)$$

$$\theta = -r_0 / r, \quad - (6)$$

then
$$\underline{r}(t) = r(t) \left(\cos \left(-\frac{r_0}{r} \right) \underline{i} + \sin \left(-\frac{r_0}{r} \right) \underline{j} \right) \quad - (7)$$

$$= r(t) \left(\cos \left(\frac{r_0}{r} \right) \underline{i} - \sin \left(\frac{r_0}{r} \right) \underline{j} \right)$$

In the non-relativistic limit the time evolution of r is

given by :

$$r = \left(\frac{L_0}{m r_0} \right) t \quad - (8)$$

so the time evolution of the vector $\underline{r}(t)$ is given by eqs. (7) and (8) as :

$$\underline{r}(t) = \left(\frac{L_0}{m\omega_0} \right) t \left(\cos \left(\frac{m\omega_0^2}{L_0 t} \right) \underline{i} - \sin \left(\frac{m\omega_0^2}{L_0 t} \right) \underline{j} \right) \quad (9)$$

so the animation proceeds as a series of (X, Y) plots as a function of time t , where:

$$X = \left(\frac{L_0}{m\omega_0} \right) t \cos \left(\frac{m\omega_0^2}{L_0 t} \right) \quad (10)$$

$$Y = - \left(\frac{L_0}{m\omega_0} \right) t \sin \left(\frac{m\omega_0^2}{L_0 t} \right) \quad (11)$$

The circular functions have a well known property:

$$-1 \leq \cos \theta \leq 1 \quad (12)$$

$$-1 \leq \sin \theta \leq 1 \quad (13)$$

so as $t \rightarrow 0 \quad (14)$

in eqs. (10) and (11):

$$X \xrightarrow{t \rightarrow 0} 0 ; Y \xrightarrow{t \rightarrow 0} 0 \quad (15)$$

$$t \rightarrow \infty \quad (16)$$

As:

$$X \rightarrow \infty \quad (17)$$

then

$$Y \rightarrow \text{constant} \quad (18)$$
