

APPLICATIONS OF ECE THEORY: THE RELATIVISTIC KINEMATICS OF ORBITS

by

M. W. Evans, H. Eckardt and R. Cheshire,

Civil List and AIAS.

(www.webarchive.org.uk, www.aias.us, www.upitec.org, www.atomicprecision.com,

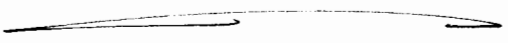
www.et3m.net.)

ABSTRACT

A relativistic theory of orbits is developed based on the Minkowski force equation, a classical limit of ECE theory. For all planar orbits the relativistic Coriolis acceleration vanishes, so the relativistic force law includes only the relativistic centripetal acceleration. The Lorentz factor is expressed in terms of the angle theta of the plane polar coordinates, allowing animations to be developed of the relativistic motion of objects in planar orbits of all kinds.

Keywords: ECE theory, relativistic kinematics of orbits, animations.

UFT 238



1. INTRODUCTION

In recent papers of this series {1 - 10} the theory of planar orbits has been developed in terms of Cartan geometry, upon which ECE theory is based as is well known. The usual approach to the theory of orbits is based {11} on classical dynamics, with adjustments from Einsteinian general relativity. In the solar system these adjustments are very small, but in objects such as whirlpool galaxies the orbits of stars around the centre of the galaxy obey neither Newtonian nor Einsteinian dynamics. In preceding papers it has been shown that the basics of whirlpool galaxies can be explained very simply with non relativistic kinematics in a plane. This reasoning results in the inference that the stars move outward from the centre, move on a hyperbolic spiral trajectory, and reach a constant velocity at infinite distance r from the centre, as observed experimentally. In Section 2 we begin the exploration of an entirely new approach to relativistic orbits based on the Minkowski force equation {11}. This is a course that relativity theory could have taken in 1905, but it followed Einstein's use of Riemann geometry, a subject that became known as general relativity. Unfortunately Einstein's mathematics are now known {1 - 10} and widely accepted {12} to be riddled with errors, notably the omission of torsion, so all inferences based on Einsteinian general relativity are meaningless. This fact has actually been known for almost a hundred years and the flaws in the Einstein field equation were discovered by Schwarzschild barely a month after Einstein's publication of perihelion precession theory in 1915 {1 - 10}. It is now widely accepted that general relativity is unscientific dogma. Obviously, it fails completely to describe the universe, because it fails completely to describe whirlpool galaxies.

The Minkowski force equation is the Newton force equation with the proper time τ replacing the time in the laboratory frame t . The equation was inferred by Minkowski shortly after Einstein's introduction of the idea of relativistic momentum. In Section 2 it is

shown that the Lorentz factor enters into the force equation via the proper time and this fact changes the force law needed to keep an object of mass m in orbit. The theory of Section 2 is developed with kinematics and so is a perfectly general theory. It can be reduced to Newtonian dynamics but never reduces to Einsteinian general relativity (EGR). This is now a requirement, because EGR is incorrect mathematically {1 - 10}. Newtonian dynamics is correct mathematically, but is conceptually lacking as is well known, for example Newtonian dynamics do not give the Coriolis force {11} and does not develop the centripetal and centrifugal forces self consistently. The reason for this is that Newtonian dynamics uses a static frame of reference, but a rotating frame of reference is needed for the Coriolis and centripetal forces and also for the orbital velocity {1 - 11}. In Section 2 it is shown that the space part of the Minkowski four-force {11} produces new and unexpected orbital properties that can be tested experimentally. These properties are graphed and animated in Section 3.

2. THE RELATIVISTIC FORCE LAW AND RELATIVISTIC ORBITS OF THE MINKOWSKI EQUATION.

Consider the relativistic velocity in plane polar coordinates:

$$\underline{v} = \frac{d\underline{r}}{d\tau} = \gamma \frac{d\underline{r}}{dt} \quad - (1)$$

where τ is the proper time and γ the Lorentz factor:

$$\gamma = \left(1 - \frac{v^2}{c^2}\right)^{-1/2} \quad - (2)$$

where c is the assumed constant speed of light in a vacuum. The relativistic acceleration is:

$$\underline{a} = \frac{d}{d\tau} \left(\frac{d\underline{r}}{d\tau} \right) = \frac{d}{d\tau} \left(\gamma \frac{d\underline{r}}{dt} \right) = \gamma \frac{d}{dt} \left(\gamma \frac{d\underline{r}}{dt} \right) \quad - (3)$$

Using the Leibnitz Theorem:

$$\underline{a} = \gamma \left(\frac{d\gamma}{dt} \frac{d\underline{r}}{dt} + \gamma \frac{d}{dt} \left(\frac{d\underline{r}}{dt} \right) \right) \quad - (4)$$

The velocity v appearing in the Lorentz factor is defined by the infinitesimal line element:

$$ds^2 = c^2 d\tau^2 = c^2 dt^2 - d\underline{r} \cdot d\underline{r} \quad - (5)$$

where:

$$d\underline{r} \cdot d\underline{r} = v^2 dt^2 \quad - (6)$$

Therefore:

$$c^2 d\tau^2 = (c^2 - v^2) dt^2 \quad - (7)$$

and the Lorentz factor is:

$$\gamma = \frac{dt}{d\tau} = \left(1 - \frac{v^2}{c^2} \right)^{-1/2} \quad - (8)$$

In plane polar coordinates:

$$d\underline{r} \cdot d\underline{r} = dr^2 + r^2 d\theta^2 \quad - (9)$$

Therefore:

$$v^2 = \left(\frac{dr}{dt} \right)^2 + r^2 \left(\frac{d\theta}{dt} \right)^2 \quad - (10)$$

In plane polar coordinates {1 - 11}:

$$\underline{r} = r \underline{e}_r \quad - (11)$$

therefore the non-relativistic velocity is {1-10}:

$$\begin{aligned} \underline{v} &= \frac{d}{dt} (r \underline{e}_r) = \frac{dr}{dt} \underline{e}_r + r \frac{d\underline{e}_r}{dt} \\ &= \frac{dr}{dt} \underline{e}_r + \omega r \underline{e}_\theta = \frac{dr}{dt} \underline{e}_r + \underline{\omega} \times \underline{r} \quad - (12) \\ &= \left(\frac{L_0}{m} \right) \left(\frac{1}{r} \underline{e}_\theta - \frac{d}{dt} \left(\frac{1}{r} \right) \underline{e}_r \right) \end{aligned}$$

where the unit vectors of the plane polar coordinate system are:

$$\underline{e}_r = \cos \theta \underline{i} + \sin \theta \underline{j} \quad - (13)$$

$$\underline{e}_\theta = -\sin \theta \underline{i} + \cos \theta \underline{j} \quad - (14)$$

as discussed in immediately preceding papers of this series on www.aiaa.us.

The non relativistic total angular momentum is a constant of motion:

$$L_0 = m r^2 \frac{d\theta}{dt} \quad - (15)$$

where the non relativistic angular velocity is:

$$\omega = \frac{d\theta}{dt} \quad - (16)$$

For a particle of mass m in orbit, its relativistic momentum is:

$$\underline{p} = \gamma m \frac{d\underline{r}}{dt} = m \frac{d\underline{r}}{d\tau} \quad - (17)$$

an equation which can be re arranged as follows:

$$p^2 c^2 = \gamma^2 m^2 c^4 \left(\frac{v}{c} \right)^2 = \gamma^2 m^2 c^4 \left(1 - \frac{1}{\gamma^2} \right) = \gamma^2 m^2 c^4 - m^2 c^4 \quad - (18)$$

giving the Einstein energy equation:

$$E^2 = c^2 p^2 + m^2 c^4 \quad (19)$$

in which

$$E = \gamma mc^2 \quad (20)$$

is the total energy, and

$$E_0 = mc^2 \quad (21)$$

is the rest energy.

The relativistic total angular momentum is:

$$L = m r^2 \frac{d\theta}{d\tau} = \gamma L_0 \quad (22)$$

and is the constant of motion L_0 multiplied by the Lorentz factor γ .

It is obvious that the concept of the Minkowski force equation uses acceleration, so this type of relativity can be used to describe the force law of orbits in a much more straightforward way than EGR, which did not consider plane polar coordinates. When Einstein was asked how he would incorporate angular motions into EGR he reply that he did not know. In fact his own energy equation is easily derived from plane polar coordinates as follows. Consider the infinitesimal line element (5), and develop it as:

$$\begin{aligned} mc^2 &= mc^2 \left(\frac{dt}{d\tau} \right)^2 - \left(\frac{dr}{d\tau} \right)^2 - r^2 \left(\frac{d\theta}{d\tau} \right)^2 \\ &= \gamma^2 mc^2 - \left(\left(\frac{dr}{d\tau} \right)^2 + r^2 \left(\frac{d\theta}{d\tau} \right)^2 \right) = \frac{E^2}{mc^2} - \frac{p^2}{c^2} \quad (23) \end{aligned}$$

So:

$$E^2 = c^2 p^2 + m^2 c^4 \quad (24)$$

which is the Einstein energy equation, Q. E. D. Note that the relativistic linear momentum in

Eq. (23) is:

$$p^2 = m^2 \left(\left(\frac{dr}{d\tau} \right)^2 + r^2 \left(\frac{d\theta}{d\tau} \right)^2 \right) \quad - (25)$$

which is Eq. (17), Q. E. D. The definition of relativistic acceleration is:

$$\underline{a} = \frac{d}{d\tau} \left(\frac{d\underline{r}}{d\tau} \right) = \gamma \left(\frac{d\gamma}{dt} \frac{d\underline{r}}{dt} + \gamma \frac{d}{dt} \left(\frac{d\underline{r}}{dt} \right) \right) \quad - (26)$$

The quantities appearing in this definition can be expressed in plane polar coordinates as

follows:

$$\frac{d\underline{r}}{dt} = \frac{dr}{dt} \underline{e}_r + \underline{\omega} \times \underline{r} \quad - (27)$$

and

$$\frac{d}{dt} \left(\frac{d\underline{r}}{dt} \right) = \frac{d^2 r}{dt^2} \underline{e}_r + \frac{d\underline{\omega}}{dt} \times \underline{r} + 2\underline{\omega} \times \frac{dr}{dt} \underline{e}_r + \underline{\omega} \times (\underline{\omega} \times \underline{r}) \quad - (28)$$

The relativistic acceleration contains more terms than the non relativistic acceleration.

Using the chain rule {11}:

$$\frac{d\gamma}{dt} = \frac{d\gamma}{dv} \frac{dv}{dt} \quad - (29)$$

where v is the velocity of the Lorentz factor defined by Eq. (10). Therefore:

$$\frac{d\gamma}{dv} = \frac{d}{dv} \left(1 - \frac{v^2}{c^2} \right)^{-1/2} = \gamma^3 \frac{v}{c^2} \quad - (30)$$

and in plane polar coordinates:

$$\begin{aligned} \underline{a} &= \gamma^4 \frac{v}{c^2} \frac{dv}{dt} \frac{d\underline{r}}{dt} + \gamma^2 \frac{d}{dt} \left(\frac{d\underline{r}}{dt} \right) \quad - (31) \\ &= \left(\frac{d\gamma}{d\tau} \frac{dr}{dt} + \gamma^2 \frac{d^2 r}{dt^2} \right) \underline{e}_r + \gamma^2 \underline{\omega} \times (\underline{\omega} \times \underline{r}) + \frac{d\gamma}{d\tau} \underline{\omega} \times \underline{r} + \gamma^2 \left(\frac{d\underline{\omega}}{dt} \times \underline{r} + 2\underline{\omega} \times \frac{dr}{dt} \underline{e}_r \right) \end{aligned}$$

In static Cartesian coordinates on the other hand:

$$\underline{a} = \frac{d}{dt} \left(\gamma \frac{dr}{dt} \right) = \gamma \frac{d\gamma}{dt} \frac{dr}{dt} + \gamma^2 \frac{d}{dt} \left(\frac{dr}{dt} \right) \quad - (32)$$

So:

$$\underline{a} (\text{Cartesian}) = \left(\gamma \frac{d\gamma}{dt} \frac{dr}{dt} + \gamma^2 \frac{d^2 r}{dt^2} \right) \underline{e}_r \quad - (33)$$

in which:

$$v = \frac{dr}{dt}, \quad \frac{d^2 r}{dt^2} = \frac{dv}{dt}, \quad \frac{d\gamma}{dv} = \gamma^3 \frac{v}{c^2} \quad - (34)$$

and

$$\frac{d\gamma}{dt} = \frac{d\gamma}{dv} \frac{dv}{dt} = \frac{\gamma^3 v}{c^2} \frac{dv}{dt} \quad - (35)$$

Therefore:

$$\underline{a} (\text{Cartesian}) = \left(\gamma^4 \frac{v^2}{c^2} + \gamma^2 \right) \frac{dv}{dt} \underline{e}_r \quad - (36)$$

in which:

$$\frac{v^2}{c^2} = 1 - \frac{1}{\gamma^2} \quad - (37)$$

and therefore:

$$\underline{a} (\text{Cartesian}) = \gamma^4 \frac{d^2 r}{dt^2} \underline{e}_r \quad - (38)$$

Using Eq. (38) in Eq. (31):

$$\underline{a} \text{ (plane polar)} = \gamma^4 \frac{d^2 \underline{r}}{dt^2} \underline{e}_r + \gamma^2 \underline{\omega} \times (\underline{\omega} \times \underline{r}) + \frac{d\gamma}{d\tau} \underline{\omega} \times \underline{r} + \gamma^2 \left(\frac{d\underline{\omega}}{dt} \times \underline{r} + 2\underline{\omega} \times \frac{d\underline{r}}{dt} \underline{e}_r \right) \quad - (39)$$

which is the expression for relativistic acceleration in plane polar coordinates.

It can be proven as follows that the relativistic Coriolis acceleration vanishes for all planar orbits. The general expression {1 - 11} for the relativistic Coriolis acceleration is:

$$\underline{a} \text{ (Coriolis)} = \gamma^2 \left(r \frac{d}{dt} \frac{d\theta}{dt} + 2 \frac{dr}{dt} \frac{d\theta}{dt} \right) \underline{e}_\theta \quad - (40)$$

in which the total relativistic angular momentum is:

$$L_0 = m r^2 \frac{d\theta}{dt} \quad - (41)$$

It follows that:

$$\frac{d}{dt} \left(\frac{d\theta}{dt} \right) = \frac{d}{dt} \left(\frac{L_0}{m r^2} \right) = \frac{d}{dr} \left(\frac{L_0}{m r^2} \right) \frac{dr}{dt} = - \frac{2 L_0}{m r^3} \frac{dr}{dt} \quad - (42)$$

so:

$$\underline{a} \text{ (Coriolis)} = \left(- \frac{2 L_0}{m r^3} \frac{dr}{dt} + \frac{2 L_0}{m r^3} \frac{dr}{dt} \right) \underline{e}_\theta = \underline{0} \quad - (43)$$

Q. E. D.

Therefore the relativistic acceleration for all planar orbits is

$$\underline{a} = \gamma^4 \frac{d^2 \underline{r}}{dt^2} \underline{e}_r + \gamma^2 \underline{\omega} \times (\underline{\omega} \times \underline{r}) + \frac{d\gamma}{d\tau} \underline{\omega} \times \underline{r} \quad - (44)$$

The relativistic centripetal component of this orbit is:

$$\underline{a}(\text{centripetal}) = \gamma^2 \underline{\omega} \times (\underline{\omega} \times \underline{r}) = -\frac{L^2}{m^2 r^3} \underline{e}_r \quad (45)$$

In Eq. (44):

$$\frac{d\gamma}{d\tau} = \gamma \frac{d\gamma}{dv} \frac{dv}{dt} = \frac{\gamma^4}{c^2} v \frac{dv}{dt} = \frac{\gamma^4}{c^2} \frac{dr}{dt} \frac{d^2 r}{dt^2} \quad (46)$$

$$\quad (47)$$

therefore the acceleration becomes:

$$\underline{a} = \gamma^4 \frac{d^2 r}{dt^2} \underline{e}_r - \frac{L^2}{m^2 r^3} \underline{e}_r + \frac{\gamma^4}{c^2} \frac{dr}{dt} \frac{d^2 r}{dt^2} \omega r \underline{e}_\theta$$

in which the relativistic total angular momentum is defined as:

$$L = \gamma L_0 = m r^2 \frac{d\theta}{d\tau} = \gamma m r^2 \omega \quad (48)$$

The relativistic force law is therefore the mass m multiplied by the relativistic acceleration:

$$\underline{a} = \left(\gamma^4 \frac{d^2 r}{dt^2} - \frac{L^2}{m^2 r^3} \right) \underline{e}_r + \frac{\gamma^4}{c^2} \frac{dr}{dt} \frac{d^2 r}{dt^2} \omega \times \underline{r} \quad (49)$$

The term in \underline{e}_θ can be developed as:

$$\underline{\omega} \times \underline{r} = \omega r \underline{e}_\theta \quad (50)$$

to arrive at:

$$\underline{a} = \left(\gamma^4 \frac{d^2 r}{dt^2} - \frac{L^2}{m^2 r^3} \right) \underline{e}_r + \frac{\gamma^4}{c^2} \frac{dr}{dt} \frac{d^2 r}{dt^2} \omega r \underline{e}_\theta \quad (51)$$

It would be very useful to transform this equation into a format from which it is possible for the relativistic force can be calculated from any observable planar orbit.

This transformation can be deduced following the method developed {1-10} in previous work for the non relativistic acceleration:

$$\underline{a} = \left(\frac{d^2 r}{dt^2} - \frac{L^2}{m^2 r^3} \right) \underline{e}_r = \frac{d^2 r}{dt^2} \underline{e}_r + \underline{\omega} \times (\underline{\omega} \times \underline{r}) \quad - (52)$$

This equation is transformed using the chain rules:

$$\frac{dr}{dt} = \frac{dr}{d\theta} \frac{d\theta}{dt} \quad - (53)$$

and

$$\frac{d}{d\theta} \left(\frac{1}{r} \right) = \frac{d}{dr} \left(\frac{1}{r} \right) \frac{dr}{d\theta} = - \frac{1}{r^2} \frac{dr}{d\theta} \quad - (54)$$

Therefore:

$$\frac{dr}{d\theta} = - r^2 \frac{d}{d\theta} \left(\frac{1}{r} \right) \quad - (55)$$

and

$$\frac{dr}{dt} = - r^2 \frac{d}{d\theta} \left(\frac{1}{r} \right) \frac{d\theta}{dt}, \quad - (56)$$

where in the non relativistic theory:

$$\omega = \frac{d\theta}{dt} = \frac{L_0}{m r^2} \quad - (57)$$

so:

$$\frac{dr}{dt} = - \frac{L_0}{m} \frac{d}{d\theta} \left(\frac{1}{r} \right) \quad - (58)$$

Similarly for any function f:

$$\frac{df}{dt} = \frac{df}{d\theta} \frac{d\theta}{dt} = \frac{L_0}{mr^2} \frac{df}{d\theta} \quad - (59)$$

If:

$$f = \frac{dr}{dt} \quad - (60)$$

then:

$$\frac{d}{dt} \left(\frac{dr}{dt} \right) = \frac{d^2 r}{dt^2} = \frac{L_0}{mr^2} \frac{d}{d\theta} \left(-\frac{L_0}{m} \frac{d}{d\theta} \left(\frac{1}{r} \right) \right) \quad - (61)$$

so:

$$\frac{d^2 r}{dt^2} = - \left(\frac{L_0}{mr} \right)^2 \frac{d^2}{d\theta^2} \left(\frac{1}{r} \right) \quad - (62)$$

It follows that the acceleration can be expressed as:

$$\underline{a} = - \left(\frac{L_0}{mr} \right)^2 \left(\frac{d^2}{d\theta^2} \left(\frac{1}{r} \right) + \frac{1}{r} \right) \underline{e}_r \quad - (63)$$

Note carefully that Eq. (63) is valid for ANY planar orbit, and that it includes the centripetal acceleration. The latter is the same for any planar orbit {1 - 10}, and the Coriolis acceleration is zero for any planar orbit.

Eq. (63) is the required equation that gives the non relativistic force for any planar orbit:

$$\underline{F} = m \underline{a} \quad - (64)$$

Q. E. D.

The relativistic counterpart of Eq. (52) is:

$$\underline{a} = \left(\gamma^4 \frac{d^2 r}{dt^2} - \frac{L^2}{m^2 r^3} \right) \underline{e}_r + \frac{\gamma^4}{c^2} \frac{dr}{dt} \frac{d^2 r}{dt^2} \underline{\omega} \times \underline{r} \quad - (65)$$

where:

$$\underline{\omega} \times \underline{r} = \omega r \underline{e}_\theta \quad - (66)$$

and in which the relativistic angular momentum:

$$L = \gamma L_0 = \gamma m r^2 \omega = \gamma m r^2 \frac{d\theta}{dt} \quad - (67)$$

must be used for self consistency. Therefore from Eq. (67):

$$\frac{d\theta}{dt} = \frac{L}{\gamma m r^2} \quad - (68)$$

so:

$$\frac{dr}{dt} = - \frac{L}{m \gamma} \frac{d}{d\theta} \left(\frac{1}{r} \right) \quad - (69)$$

Similarly Eq. (59) becomes:

$$\frac{df}{dt} = \frac{df}{d\theta} \frac{d\theta}{dt} = \frac{L}{\gamma m r^2} \frac{df}{d\theta} \quad - (70)$$

so Eq. (62) becomes:

$$\frac{d^2 r}{dt^2} = - \left(\frac{L}{\gamma m r} \right)^2 \frac{d^2}{d\theta^2} \left(\frac{1}{r} \right) \quad - (71)$$

Using Eq. (71) in Eq. (65):

$$\underline{a} = - \left(\left(\frac{\gamma L}{m r} \right)^2 \frac{d^2}{d\theta^2} \left(\frac{1}{r} \right) + \frac{L^2}{m^2 r^3} \right) \underline{e}_r + \frac{\gamma^4}{c^2} \frac{dr}{dt} \frac{d^2 r}{dt^2} \omega r \underline{e}_\theta$$

and using Eqs. (68), (69) and (71):

$$\underline{a} = - \left(\frac{L}{m r} \right)^2 \left(\gamma^2 \frac{d^2}{d\theta^2} \left(\frac{1}{r} \right) + \frac{1}{r} \right) \underline{e}_r + \frac{L^4}{m^4 c^2 r^3} \frac{d}{d\theta} \left(\frac{1}{r} \right) \frac{d^2}{d\theta^2} \left(\frac{1}{r} \right) \underline{e}_\theta \quad - (73)$$

This is the required equation that gives the relativistic force produced by any planar orbit:

$$\underline{F} = m \underline{a} \quad - (74)$$

Q. E. D.

For the purposes of graphics and animation in Section 3 of this paper it is convenient to express the Lorentz factor in terms of the angle θ . In order to achieve this aim consider the non relativistic velocity of the Lorentz factor in plane polar coordinates

$$\underline{v} = \frac{dr}{dt} \underline{e}_r + \omega r \underline{e}_\theta \quad - (75)$$

As in preceding papers

$$\underline{v} = \left(\frac{L_0}{m} \right) \left(\frac{1}{r} \underline{e}_\theta - \frac{d}{d\theta} \left(\frac{1}{r} \right) \underline{e}_r \right) \quad - (76)$$

For an elliptical orbit:

$$\frac{1}{r} = \frac{1}{d} (1 + \epsilon \cos \theta) \quad - (77)$$

so:

$$\frac{d}{d\theta} \left(\frac{1}{r} \right) = - \frac{\epsilon}{d} \sin \theta \quad - (78)$$

and the non relativistic velocity is:

$$\underline{v} = \left(\frac{L_0}{md} \right) \left((1 + \epsilon \cos \theta) \underline{e}_\theta + \epsilon \sin \theta \underline{e}_r \right) \quad - (79)$$

Therefore:

$$v^2 = \left(\frac{L_0}{md} \right)^2 (1 + \epsilon^2 + 2\epsilon \cos \theta) \quad - (80)$$

which is the required expression for v of the Lorentz factor in terms of θ , Q. E. D. Such a

procedure can be repeated for any planar orbit in which the dependence of r on θ is known.

The non relativistic velocity v can be expressed in terms of ω and r using Eqs.

(75), (76) and the non relativistic total angular momentum:

$$L_0 = mr^2\omega = mr^2 \frac{d\theta}{dt} \quad - (81)$$

For any planar orbit:

$$\underline{v} = \omega r^2 \left(\frac{1}{r} \underline{e}_\theta - \frac{d}{d\theta} \left(\frac{1}{r} \right) \underline{e}_r \right) \quad - (82)$$

so:

$$v^2 = \omega^2 r^2 \left(1 + r^2 \left(\frac{d}{d\theta} \left(\frac{1}{r} \right) \right)^2 \right) \quad - (83)$$

which is the required expression, Q. E. D. For the elliptical orbit for example:

$$\cos \theta = \frac{1}{\epsilon} \left(\frac{d}{r} - 1 \right) \quad - (84)$$

so using Eqs. (83) and (84):

$$v^2 = \frac{r^4 \omega^2}{d^2} \left(1 + \epsilon^2 + 2 \left(\frac{d}{r} - 1 \right) \right) \quad - (85)$$

The non relativistic orbital linear velocity of the elliptical orbit is:

$$v^2 = \frac{d\underline{r} \cdot d\underline{r}}{dt^2} \quad - (86)$$

and this is used in the Lorentz factor. In the case of a circular orbit:

$$\epsilon = 0, \quad d = r \quad - (87)$$

and the familiar result is obtained that for a circular orbit in non relativistic theory:

$$v = \omega r. \quad - (88)$$

In summary the relativistic force for any planar orbit is defined by:

-(89)

$$\underline{F} = -\frac{L^2}{mr^3} \left(\gamma^2 \frac{d^2}{dt^2} \left(\frac{1}{r} \right) + \frac{1}{r} \right) \underline{e}_r + \frac{L^4}{m r^3 c^3} \frac{d}{dt} \left(\frac{1}{r} \right) \frac{d^2}{dt^2} \left(\frac{1}{r} \right) \underline{e}_\theta$$

in which the Lorentz factor is:

$$\gamma = \left(1 - \left(\frac{L_0}{mc} \right)^2 \left(\frac{1}{r^2} + \left(\frac{d}{dt} \left(\frac{1}{r} \right) \right)^2 \right) \right)^{-1/2} \quad -(90)$$

and in which the relativistic total angular momentum is:

$$L = \gamma L_0 = \gamma m r^2 \frac{d\theta}{dt} = \gamma m r^2 \omega \quad -(91)$$

Some force laws of various orbits are discussed in Section 3.

For the purposes of animation (Section 3) the analytical expressions for the time evolution is needed for an orbiting object of mass m . In the non relativistic theory of orbits the time evolution is calculated {1 - 11} from the total angular momentum:

$$L_0 = m r^2 \frac{d\theta}{dt} \quad -(92)$$

Therefore:

$$\frac{d\theta}{dt} = \frac{L_0}{m r^2} \quad -(93)$$

In general the planar orbit is:

$$r = f(\theta) \quad -(94)$$

Therefore:

$$dt = \frac{m}{L_0} f^2(\theta) d\theta \quad -(95)$$

which integrates to:

$$t = \int dt = \frac{m}{L_0} \int f^2(\theta) d\theta \quad - (96)$$

For the elliptical orbit:

$$f(\theta) = \frac{d}{1 + \epsilon \cos \theta} \quad - (97)$$

and for the hyperbolic spiral

$$f(\theta) = -\frac{r_0}{\theta} \quad - (98)$$

Using the chain rule:

$$\frac{d\theta}{dt} = \frac{dr}{dt} \frac{d\theta}{dr} = \frac{L_0}{mr^2} \quad - (99)$$

it is found that:

$$\frac{dr}{dt} = \frac{L_0}{mr^2} \frac{dr}{d\theta} = -\frac{L_0}{m} \frac{d}{d\theta} \left(\frac{1}{r} \right) \quad - (100)$$

For the hyperbolic spiral (98):

$$\frac{d}{d\theta} \left(\frac{1}{r} \right) = -\frac{1}{r_0} \quad - (101)$$

therefore:

$$r = \left(\frac{L_0}{mr_0} \right) t \quad - (102)$$

The relativistic equivalent of Eq. (93) is:

$$\frac{d\theta}{d\tau} = \gamma \frac{L_0}{mr^2} \quad - (103)$$

where

$$\gamma = \frac{dt}{d\tau} = \left(1 - \frac{v^2}{c^2}\right)^{-1/2} \quad - (104)$$

and

$$v^2 = \left(\frac{dr}{dt}\right)^2 + r^2 \left(\frac{d\theta}{dt}\right)^2 = \left(\frac{dr}{dt}\right)^2 \left(1 + r^2 \left(\frac{d\theta}{dr}\right)^2\right) \quad - (105)$$

For the hyperbolic spiral (98):

$$\left(\frac{dr}{dt}\right)^2 = \left(\frac{L_0}{mr_0}\right)^2 \quad - (106)$$

and

$$\left(\frac{d\theta}{dr}\right)^2 = \frac{r_0^2}{r^4} \quad - (107)$$

so the velocity of the Lorentz factor is given by:

$$v^2 = \left(\frac{L_0}{mr_0}\right)^2 \left(1 + \left(\frac{r_0}{r}\right)^2\right) \quad - (108)$$

from which it is found that:

$$v \xrightarrow{r \rightarrow \infty} \frac{L_0}{mr_0} \quad - (109)$$

as observed experimentally in the well known velocity curve of the whirlpool galaxy.

Therefore the Lorentz factor is:

$$\gamma = \left(1 - \left(\frac{L_0}{mrc}\right)^2 \left(1 + \left(\frac{r_0}{r}\right)^2\right)\right)^{-1/2} \quad - (110)$$

Using the chain rules:

$$\frac{d\theta}{d\tau} = \frac{dr}{d\tau} \frac{d\theta}{dr} = \gamma \frac{L_0}{mr^2} \quad - (111)$$

and

$$\frac{dr}{d\tau} = \frac{\gamma L_0}{m r^2} \cdot \frac{dr}{dt} \quad - (112)$$

it is found that:

$$r = \frac{\gamma L_0}{m r_0} \tau \quad - (113)$$

In the frame of the proper time τ , r is proportional to $\gamma\tau$. The proper time is the time measured in the frame in which the particle is at rest. The observer or laboratory time t is the time in the frame with respect to which the particle of mass m is moving. By definition of the Lorentz transform:

$$\gamma = \frac{dt}{d\tau} \quad - (114)$$

so

$$\tau = \frac{t}{\gamma} \quad - (115)$$

and

$$\tau = \left(1 - \frac{v^2}{c^2}\right)^{1/2} t \quad - (116)$$

Therefore from Eqs. (103), (110) and (113):

$$r = \left(1 - \left(\frac{L_0}{m r_0 c}\right)^2 \left(1 + \left(\frac{r_0}{r}\right)^2\right)\right)^{1/2} \frac{L_0}{m r_0} \tau \quad - (117)$$

and in the relativistic theory of the hyperbolic spiral orbit there is no longer a simple dependence of r on t . Solving for r (see note 238(9)):

$$r = \frac{\tau}{\sqrt{2}} \left(A\tau + (A^2\tau^2 - 4B)^{1/2} \right)^{1/2} \quad - (118)$$

where:

$$A = \left(1 - \left(\frac{L_0}{m r_0 c}\right)^2\right) \left(\frac{L_0}{m r_0}\right)^2 - (119)$$

and

$$B = \left(\frac{L_0}{m r_0 c}\right)^2 \left(\frac{L_0}{m r_0}\right)^2 r_0^2 - (120)$$

This equation reduces to Eq. (102) in the non relativistic limit $v \ll c$.

In order to animate the motion of a mass m along a hyperbolic spiral trajectory it is convenient to transform coordinates. Starting from:

$$\underline{r}(t) = r(t) \underline{e}_r - (121)$$

express the unit vector as:

$$\underline{e}_r = \cos \theta \underline{i} + \sin \theta \underline{j} - (122)$$

so:

$$\underline{r}(t) = r(t) (\cos \theta \underline{i} + \sin \theta \underline{j}) - (123)$$

For the hyperbolic spiral:

$$r = -r_0 / \theta, - (124)$$

$$\theta = -r_0 / r, - (125)$$

so:

$$\begin{aligned} \underline{r}(t) &= r(t) \left(\cos\left(-\frac{r_0}{r}\right) \underline{i} + \sin\left(-\frac{r_0}{r}\right) \underline{j} \right) \\ &= r(t) \left(\cos\left(\frac{r_0}{r}\right) \underline{i} - \sin\left(\frac{r_0}{r}\right) \underline{j} \right) - (126) \end{aligned}$$

In the non relativistic limit:

$$r = \left(\frac{L_0}{m r_0} \right) t \quad - (127)$$

so:

$$\underline{r}(t) = \left(\frac{L_0}{m r_0} \right) t \left(\cos \left(\frac{m r_0^2}{L_0 t} \right) \underline{i} - \sin \left(\frac{m r_0^2}{L_0 t} \right) \underline{j} \right) \quad - (128)$$

The animation proceeds as a series of (X, Y) plots as a function of time t, where:

$$X = \left(\frac{L_0}{m r_0} \right) t \cos \left(\frac{m r_0^2}{L_0 t} \right), \quad - (129)$$

$$Y = \left(\frac{L_0}{m r_0} \right) t \sin \left(\frac{m r_0^2}{L_0 t} \right). \quad - (130)$$

The circular functions have the well known property:

$$-1 \leq \cos \theta \leq 1, \quad - (131)$$

$$-1 \leq \sin \theta \leq 1, \quad - (132)$$

so as

$$t \rightarrow 0 \quad - (133)$$

in Eqs. (10) and (11):

$$X \xrightarrow[t \rightarrow 0]{} 0, \quad Y \xrightarrow[t \rightarrow 0]{} 0 \quad - (134)$$

and as

$$t \rightarrow \infty \quad - (135)$$

then:

$$X \rightarrow \infty \quad - (136)$$

$$Y \rightarrow \text{constant}, \quad - (137)$$

and the particle moves outward along the spiral from the centre of the galaxy.

To animate the relativistic motion of a mass m along the hyperbolic spiral use the relativistic Eq. (118) in Eq. (126).

The animation of the motion of a mass m along the elliptical orbit is a non trivial problem analytically but can be coded with the following method. In this case the non relativistic theory gives:

$$t = \int dt = \frac{m d^2}{L_0} \int \frac{d\theta}{(1 + \epsilon \cos \theta)^2} \quad - (138)$$

$$= \frac{m d^2}{L_0 (1 - \epsilon^2)} \left[\frac{2}{(1 - \epsilon^2)^{1/2}} \tan^{-1} \left(\frac{(1 - \epsilon) \tan(\theta/2)}{(1 - \epsilon^2)^{1/2}} \right) - \frac{\epsilon \sin \theta}{1 + \epsilon \cos \theta} \right] \quad - (139)$$

and as is well known [11] this equation cannot be inverted analytically to find θ as a function of time t . The trajectory as a function of θ is:

$$\underline{r}(\theta) = r(\theta) (\cos \theta \underline{i} + \sin \theta \underline{j}) \quad - (140)$$

where:

$$r(\theta) = \frac{d}{1 + \epsilon \cos \theta} \quad - (141)$$

Therefore:

$$X = \frac{d \cos \theta}{1 + \epsilon \cos \theta}, \quad Y = \frac{d \sin \theta}{1 + \epsilon \cos \theta} \quad - (142)$$

and plot the points (X, Y) as a function of θ (see note 238(12)). The points (X, Y) can be plotted as a function of t as required for an animation by using Eq. (139) to transform θ into t . For example:

$$t_1 = \frac{md^2}{L_0} \int_0^{\theta_1} \frac{d\theta}{(1 + \epsilon \cos \theta)^2} \quad - (143)$$

$$= \frac{md^2}{L_0(1 - \epsilon^2)} \left[\frac{2}{(1 - \epsilon^2)^{1/2}} \tan^{-1} \left(\frac{(1 - \epsilon) \tan(\theta/2)}{(1 - \epsilon^2)^{1/2}} \right) - \frac{\epsilon \sin \theta}{1 + \epsilon \cos \theta} \right]$$

This method gives the correct speed and obeys the three Kepler laws automatically.

For the precessing ellipse or conical section the non relativistic theory gives:

$$r = \frac{d}{1 + \epsilon \cos(x\theta)} \quad - (144)$$

and:

$$t = \frac{md^2}{L_0} \int \frac{d\theta}{(1 + \epsilon \cos(x\theta))^2} \quad - (145)$$

so:

$$X = \frac{d \cos \theta}{1 + \epsilon \cos(x\theta)}, \quad Y = \frac{d \sin \theta}{1 + \epsilon \cos(x\theta)} \quad - (146)$$

and plot (X, Y) as a function of time t. It is known from previous work {1 - 10} that fractal conical sections emerge when x is increased, and the animation of this type of orbit will give very interesting results in mathematics and astronomy.

The relativistic theory of the conical section orbits gives:

$$\frac{d\theta}{d\tau} = \gamma \frac{L_0}{mr^2} \quad - (147)$$

in which the Lorentz factor is:

$$\gamma = \frac{dt}{d\tau} = \left(1 - \frac{v^2}{c^2} \right)^{-1/2} \quad - (148)$$

with velocity defined by:

$$\underline{v} = \frac{L_0}{m} \left(\frac{1}{r} e_\theta - \frac{d}{d\theta} \left(\frac{1}{r} \right) e_r \right) \quad (149)$$

From Eq. (147):

$$d\tau = \left(1 - \frac{v^2}{c^2} \right)^{1/2} \frac{m r^2}{L_0} d\theta \quad (150)$$

and integrating gives:

$$\tau = \frac{m}{L_0} \int \left(1 - \frac{v^2}{c^2} \right)^{1/2} r^2 d\theta \quad (151)$$

For the conical sections Eq. (80) gives the velocity of the Lorentz factor as a function of θ as:

$$v^2 = \left(\frac{L_0}{m d} \right)^2 \left(1 + e^2 + 2e \cos \theta \right) \quad (152)$$

so:

$$\tau = \frac{m d^2}{L_0} \int \left(1 - \left(\frac{L_0}{m d c} \right)^2 \left(1 + e^2 + 2e \cos \theta \right) \right)^{1/2} \frac{d\theta}{(1 + e \cos \theta)^2} \quad (153)$$

which must be integrated numerically. The animation proceeds by using Eq. (153) and

calculating τ for each θ . For example:

$$\tau_1 = \frac{m}{L_0} \int_0^{\theta_1} \left(1 - \left(\frac{L_0}{m d c} \right)^2 \left(1 + e^2 + 2e \cos \theta \right) \right)^{1/2} \frac{d\theta}{(1 + e \cos \theta)^2} \quad (154)$$

Repeat for t_2, t_3, \dots, t_n and animate (X, Y) as a function of t.

Finally the relativistic theory of the precessing conical sections proceeds with

$$r = \frac{d}{1 + \epsilon \cos(x\theta)} \quad - (155)$$

and:

$$\tau = \frac{m d^2}{L_0} \int \left(1 - \frac{v^2}{c^2}\right)^{1/2} \frac{d\theta}{(1 + \epsilon \cos(x\theta))^2} \quad - (156)$$

The velocity of the Lorentz factor is:

$$\underline{v} = \frac{L_0}{m} \left(\frac{(1 + \epsilon \cos(x\theta))}{d} \underline{e}_\theta - \frac{d}{d\theta} \left(\frac{1}{r} \right) \underline{e}_r \right) \quad - (157)$$

where:

$$\frac{d}{d\theta} \left(\frac{1}{r} \right) = - \frac{x\epsilon}{d} \sin(x\theta) \quad - (158)$$

so

$$v^2 = \left(\frac{L_0}{m d} \right)^2 \left((1 + \epsilon \cos(x\theta))^2 + x^2 \epsilon^2 \sin^2(x\theta) \right) \quad - (159)$$

Therefore τ is computed from Eqs. (156) and (159) for a given θ , for example:

$$\tau_1 = \frac{m d^2}{L_0} \int_0^{\theta_1} \left(1 - \frac{v^2}{c^2}\right)^{1/2} \frac{d\theta}{(1 + \epsilon \cos(x\theta))^2} \quad - (160)$$

with:

$$v^2 = \left(\frac{L_0}{m d} \right)^2 \left((1 + \epsilon \cos(x\theta))^2 + x^2 \epsilon^2 \sin^2(x\theta) \right) \quad - (161)$$

3. DISCUSSION OF GRAPHICAL RESULTS AND ANIMATIONS

ACKNOWLEDGMENTS

The British Government is thanked for a Civil List Pension and the AIAS and others for discussions. Dave Burleigh is thanked for posting and Alex Hill and Robert Cheshire for translations and broadcasting. AIAS is established in the Trust Law of England and Wales under the Newlands Family Trust (Est. 2012). It is closely associated with UPITEC, which is a not for profit organization in Boise, Idaho, U. S. A.

REFERENCES

- {1} M. W. Evans, Ed. "Definitive Refutations of the Einsteinian General Relativity" (Cambridge International Science Publishing, CISP, www.cisp-publishing.com, 2012, special issue six of ref. (2)).
- {2} M. W. Evans, Ed., Journal of Foundations of Physics and Chemistry (CISP from June 2011, six issues a year).
- {3} M. W. Evans, S. J. Crothers, H. Eckardt and K. Pendergast, "Criticisms of the Einstein Field Equation" (CISP, 2011).
- {4} M. W. Evans, H. Eckardt and D. W. Lindstrom, "Generally Covariant Unified Field Theory" (Abramis, 2005 to 2011) in seven volumes.
- {5} L. Felker, "The Evans Equations of Unified Field Theory" (Abramis 2007, translated into Spanish by Alex Hill on www.aias.us).
- {6} M. W. Evans, H. Eckardt and D. W. Lindstrom, papers and plenary paper in the Serbian Academy of Sciences. Also Found. Phys. Lett. and Physica B.
- {7} M. W. Evans and L. B. Crowell, "Classical and Quantum Electrodynamics and the B(3) Field" (World Scientific 2001).
- {8} M. W. Evans and S. Kielich, Eds., "Modern Nonlinear Optics" (Wiley Interscience, New York, 1992, 1993, 1997 and 2001) in six volumes and two editions.

{9} M. W. Evans and J.-P. Vigiier, "The Enigmatic Photon" (Kluwer, Dordrecht, 1994 to 2002), in ten volumes hardback and softback.

{10} M. W. Evans and A. A. Hasanein, "The Photomagnetron in Quantum Field Theory" (World Scientific 1994).

{11} J. B. Marion and S. T. Thornton, "Classical Dynamics" (Harcourt, 3rd. Edition, 1988).

{12} Feedback to the ECE sites indicating thirty million readings in the past decade, and the archived feedback data.