# 231(7): Definition of Potential Energy and Reduction to Schroedinger Equation

In the last note it was concluded that:

$$\epsilon^2 - c^2\pi^2 = E^2 - c^2p^2 = m^2c^4 \quad -(1)$$

which is a frame transformation induced by:

$$\underline{e}^{(1)} \rightarrow \underline{e}^{(1)} e^{i\phi} \quad -(2)$$

$$\underline{e}^{(2)} \rightarrow \underline{e}^{(2)} e^{-i\phi}, \quad -(3)$$

$$\phi = \omega t - \kappa Z. \quad -(4)$$

Therefore:

$$\epsilon^2 = E^2 - c^2(p^2 - \pi^2). \quad -(5)$$

In the minimal prescription:

$$\epsilon = E - V \quad -(6)$$

where $V$ is a potential energy of spacetime introduced by this frame transformation. From eqs. (5) and (6):

$$(E - V)^2 = E^2 + V^2 - 2EV \quad -(7)$$

$$= E^2 - c^2(p^2 - \pi^2).$$

In the approximation:

$$E \rightarrow mc^2 \quad -(8)$$

eq. (7) produces:

$$V^2 - 2mc^2V + c^2(p^2 - \pi^2) = 0$$

$$-(9)$$

2) for which:

$$V = 2mc^2\left(1 \pm \left(1 - \frac{(p^2 - \pi^2)}{m^2c^2}\right)^{1/2}\right) \quad -(10)$$

In the limit: $\quad p = \pi \quad -(11)$

there is no potential energy, so the negative root is chosen in eq. (10) to give:

$$\boxed{V = 2mc^2\left(1 - \left(1 - \frac{(p^2 - \pi^2)}{m^2c^2}\right)^{1/2}\right)} \quad -(12)$$

given the approximation (8)

In eq. (12):

$$p^2 = p_x^2 + p_y^2 + p_z^2 \quad -(13)$$

$$\pi^2 = p_x^2 e^{i\phi} + p_y^2 e^{-i\phi} + p_z^2 \quad -(14)$$

so:

$$p^2 - \pi^2 = p_x^2(1 - e^{i\phi}) + p_y^2(1 - e^{-i\phi}) \quad -(15)$$

$$\mathrm{Real}(p^2 - \pi^2) = p_x^2(1 - \cos\phi) + p_y^2(1 - \cos\phi)$$

$$= (p_x^2 + p_y^2)(1 - \cos\phi), \quad -(16)$$

$$\phi = \omega t - \kappa Z. \quad (17)$$

3) Therefore in eq. (1):

$$(E - V)^2 - m^2c^4 = c^2\pi^2 \quad -(18)$$

i.e.

$$E - V - mc^2 = \frac{c^2\pi^2}{E - V + mc^2} \quad -(19)$$

In the approximation (8):

$$E - mc^2 = V + \frac{c^2\pi^2}{2mc^2 - V} \quad -(20)$$

If it is assumed that:

$$V << 2mc^2 \quad -(21)$$

then

$$E - mc^2 = \frac{1}{2m}\pi^2 + V \quad -(22)$$

Finally for

$$\pi \sim p \quad -(23)$$

$$E - mc^2 = \frac{p^2}{2m} + V \quad -(24)$$

The Schroedinger equation is obtained by regarding the energy as $E - mc^2$, so:

$$E\phi = -\frac{\hbar^2}{2m}\nabla^2\phi + V\phi \quad -(25)$$

$$V = 2mc^2\left(1 - \left(1 - \frac{(p_x^2 + p_y^2)(1 - \cos\phi)}{m^2c^2}\right)\right)^{1/2}$$