231(1): Development of the Tetrad in Terms of the Metric.

Consider for simplicity the metric tensor in three dimensional space:

$$g_{ij} = g_{ji} = h_i h_j \underline{e}_i \cdot \underline{e}_j \quad -(1)$$

$$= \frac{\partial \underline{r}}{\partial u_i} \cdot \frac{\partial \underline{r}}{\partial u_j}$$

in curvilinear coordinates. Here:

$$\frac{\partial \underline{r}}{\partial u_i} = h_i \underline{e}_i \quad -(2)$$

where $\underline{e}_i$ are the unit vectors, and where the scaling factors are

$$h_i = \left| \frac{\partial \underline{r}}{\partial u_i} \right| . \quad -(3)$$

In the Cartesian coordinate system:

$$\underset{\sim}{r} = X \underline{i} + Y \underline{j} + Z \underline{k}, \quad -(4)$$

the unit vectors are:

$$\underline{i} = \frac{\partial \underline{r}}{\partial X}, \quad \underline{j} = \frac{\partial \underline{r}}{\partial Y}, \quad \underline{k} = \frac{\partial \underline{r}}{\partial Z} . \quad -(5)$$

The scaling factors are:

$$h_1 = |\underline{i}| = (\underline{i} \cdot \underline{i})^{1/2} = 1 \quad -(6)$$

and

$$h_2 = h_3 = 1 \quad -(7)$$

So the metric tensor is:

$$g_{ij} = g_{ji} = \underline{e}_i \cdot \underline{e}_j \quad -(8)$$

2) i.e.

$$g_{ij} = g_{ji} = \begin{bmatrix} \underline{i}\cdot\underline{i} & \underline{i}\cdot\underline{j} & \underline{i}\cdot\underline{k} \\ \underline{j}\cdot\underline{i} & \underline{j}\cdot\underline{j} & \underline{j}\cdot\underline{k} \\ \underline{k}\cdot\underline{i} & \underline{k}\cdot\underline{j} & \underline{k}\cdot\underline{k} \end{bmatrix} \quad -(9)$$

$$= \begin{bmatrix} \underline{i} \\ \underline{j} \\ \underline{k} \end{bmatrix} \cdot \begin{bmatrix} \underline{i} & \underline{j} & \underline{k} \end{bmatrix} \quad -(10)$$

In flat space:

$$g_{ij} = g_{ji} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad -(11)$$

In general:

$$g^{\mu\nu} = \underline{e}^{\mu} \cdot \underline{e}^{\nu T}. \quad -(12)$$

If the tetrad is defined by:

$$\underline{e}^{(a)} = q^{(a)}_{\mu}\, \underline{e}^{\mu} \quad -(13)$$

then:

$$\underline{e}^{(a)} \cdot \underline{e}^{\nu T} = q^{(a)}_{\mu}\, \underline{e}^{\mu} \cdot \underline{e}^{\nu T} \quad -(14)$$

i.e.

$$\boxed{g^{a\nu} = q^{a}_{\mu}\, g^{\mu\nu}} \quad -(15)$$

and

$$\boxed{q^{a}_{\mu} = g^{a\nu} g_{\mu\nu}} \quad -(16)$$

3) because
$$g^{\mu\nu} g_{\mu\nu} = 1 \quad -(17)$$

If a or (a) represents the complex circular basis then:
$$g^{a\nu} = \underline{e}^{a} \cdot \underline{e}^{\nu T} \quad -(18)$$

and
$$q^{a}_{\mu} = \underline{e}^{a} \cdot \underline{e}^{\nu T} g_{\mu\nu} \quad -(19)$$

with
$$g_{\mu\nu} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad -(20)$$

So the space parts of q^{a}_{μ} are given by:
$$q^{(a)}_{\mu} = \begin{bmatrix} \underline{e}^{(1)} \cdot \underline{i} & \underline{e}^{(1)} \cdot \underline{j} & \underline{e}^{(1)} \cdot \underline{k} \\ \underline{e}^{(2)} \cdot \underline{i} & \underline{e}^{(2)} \cdot \underline{j} & \underline{e}^{(2)} \cdot \underline{k} \\ \underline{e}^{(3)} \cdot \underline{i} & \underline{e}^{(3)} \cdot \underline{j} & \underline{e}^{(3)} \cdot \underline{k} \end{bmatrix} \quad (21)$$

For example there are tetrad components such as:
$$q^{(1)}_{1} = \underline{e}^{(1)} \cdot \underline{i} \quad -(22)$$
$$= \underline{e}^{(1)} \cdot \underline{e}^{1}$$

the tetrad postulate asserts that:
$$D_\nu q^{a}_{\mu} = 0 \quad -(23)$$

so
$$D_\nu q^{(1)}_{1} = 0. \quad -(24)$$

for all ν.

4)

When $\underline{e}^{(1)}$ and $\underline{i}$ are static unit vectors eq. (24) is a trivial result, because:

$$\underline{e}^{(1)} = \frac{1}{\sqrt{2}} \left(\underline{i} - i\underline{j} \right) = \text{constant}, \quad -(25)$$

$$\underline{i} = \text{constant}. \quad -(26)$$

However, in the case of circularly polarized dynamics, $\underline{e}^{(1)}$ is phase dependent, as in previous UFT papers:

$$\underline{e}^{(1)} = \frac{1}{\sqrt{2}} \left(\underline{i} - i\underline{j} \right) \exp\left(-i\left(\omega t - \underline{\kappa} \cdot \underline{Z} \right) \right), \quad -(27)$$

and its complex conjugate is:

$$\underline{e}^{(2)} = \frac{1}{\sqrt{2}} \left(\underline{i} + i\underline{j} \right) \exp\left(i\left(\omega t - \underline{\kappa} \cdot \underline{Z} \right) \right) \quad -(28)$$

with:

$$\underline{e}^{(3)} = \underline{k}, \quad -(29)$$

so

$$\underline{e}^{(1)} \times \underline{e}^{(2)} = i\,\underline{e}^{(3)*} \quad -(30)$$

et cyclicum.

This is a frame of reference that rotates and translates along the Z axis.

It must therefore be defined by a spin connection and gamma connection. The tetrad postulate is:

$$D_\nu q^a_\mu = \partial_\nu q^a_\mu + \omega^a_{\nu b} q^b_\mu - \Gamma^\lambda_{\mu\nu} q^a_\lambda = 0 \quad -(31)$$

$$= \partial_\nu q^a_\mu + \omega^a_{\nu\mu} - \Gamma^a_{\nu\mu}$$

5) Therefore:

$$d_\nu q^a_\mu = \Omega^a_{\ \nu\mu} \quad -(32)$$

where

$$\Omega^a_{\ \nu\mu} = \Gamma^a_{\ \nu\mu} - \omega^a_{\ \nu\mu}. \quad -(33)$$

For two static frames eqs. (25) and (26):

$$\Omega^a_{\ \mu\nu} = 0. \quad -(34)$$

However, from eq. (27) in eq. (22):

$$q^{(1)}_1 = \frac{1}{\sqrt{2}} \exp\left(-i\left(\omega t - \underline{\kappa} \cdot \underline{Z}\right)\right) \quad -(35)$$

and:

$$\frac{1}{c}\frac{\partial q^{(1)}_1}{\partial t} = -i\frac{\omega}{c} q^{(1)}_1 = \Omega^{(1)}_{\ 01}, \quad -(36)$$

$$\frac{\partial q^{(1)}_1}{\partial Z} = i\kappa q^{(1)}_1 = \Omega^{(1)}_{\ 31}. \quad -(37)$$

There are two spin connection components:

$$\Omega^{(1)}_{\ 01} = -\frac{i}{\sqrt{2}}\frac{\omega}{c} \exp\left(-i\left(\omega t - \underline{\kappa} \cdot \underline{Z}\right)\right) \quad -(38)$$

and

$$\Omega^{(1)}_{\ 31} = \frac{i}{\sqrt{2}} \kappa \exp\left(-i\left(\omega t - \underline{\kappa} \cdot \underline{Z}\right)\right) \quad -(39)$$

For propagation at the speed of light:

6)

$$\kappa = \frac{\omega}{c} \quad -(40)$$

So

$$\Omega^{(1)}_{01} = -\Omega^{(1)}_{31} . \quad -(41)$$

From eq. (32):

$$\partial^\nu \partial_\nu q^a_\mu = \Box q^a_\mu = \partial^\nu \Omega^a_{\nu\mu} . \quad -(42)$$

Define:

$$-R q^a_\mu := \partial^\nu \Omega^a_{\nu\mu} , \quad -(43)$$

then

$$(\Box + R) q^a_\mu = 0 \quad -(44)$$

which is the ECE wave equation with:

$$R := - q^\mu_a \partial^\nu \Omega^a_{\nu\mu} . \quad -(45)$$

For example:

$$(\Box + R) q^{(1)}_1 = 0 \quad -(46)$$

where

$$\Box = \frac{1}{c^2} \frac{\partial^2}{\partial t^2} - \nabla^2 \quad -(47)$$

and

$$q^{(1)}_1 = \frac{1}{\sqrt{2}} \exp\left(-i \left(\omega t - \underline{\kappa} \cdot \underline{Z} \right) \right) \quad -(48)$$

It follows that

$$R = \frac{\omega^2}{c^2} + \kappa^2 . \quad -(49)$$

It can be checked that:

7)

$$\Box q^{(1)}_{1} = \partial^{\nu} \Omega^{(1)}_{\nu 1}$$
$$= \partial^{0} \Omega^{(1)}_{01} + \partial^{3} \Omega^{(1)}_{31}$$
$$= \frac{1}{c} \frac{\partial \Omega^{(1)}_{01}}{\partial t} - \frac{\partial \Omega^{(1)}_{31}}{\partial Z}, \quad -(50)$$

where

$$\Omega^{(1)}_{01} = -\frac{i\omega}{c} q^{(1)}_{1}, \quad -(51)$$
$$\Omega^{(1)}_{31} = ik\, q^{(1)}_{1}, \quad -(52)$$

so

$$\partial^{0} \Omega^{(1)}_{01} = -\frac{i\omega}{c} \partial^{0} q^{(1)}_{1} = -\frac{\omega^2}{c^2} q^{(1)}_{1} \quad -(53)$$
$$\partial^{3} \Omega^{(1)}_{31} = ik\, \partial^{3} q^{(1)}_{1} = -k^2 q^{(1)}_{1} \quad -(54)$$

so

$$\boxed{\Box q^{(1)}_{1} = \partial^{\nu} \Omega^{(1)}_{\nu 1} = -R\, q^{(1)}_{1}} \quad -(55)$$

Q.E.D.

In general the wave equation of ECE can be derived from the definition (16):

$$q^{a}_{\mu} = g^{a\nu} g_{\mu\nu} = g^{a}_{\mu} \quad -(56)$$

which shows that the tetrad is a mixed index metric. Most generally, a and μ may indicate

8) two different mathematical spaces, or two different representations of the same mathematical space. Using the definition (56) the metric compatibility condition and the tetrad postulate become the same:

$$\boxed{D_\nu q^a_\mu = D_\nu g^a_\mu = 0} \quad -(57)$$

The connection may be defined in term of the mixed index metric as:

$$\boxed{\Omega^a_{\nu\mu} = \partial_\nu g^a_\mu} \quad -(58)$$

and the wave equation is:

$$\boxed{(\square + R)\, g^a_\mu = 0} \quad -(59)$$

So the eigenfunctions of the wave equations of physics are defined as mixed index metrics.

The mixed index metric g^a_μ is the mixed index metric $g^{a\nu}$ with index lowered by the metric $g_{\mu\nu}$. Finally in this note the Cartan structure equations and identity apply with q^a_μ replaced by g^a_μ.