216(8) : The Gravitational Lennard – Jones Potential.

Strictly speaking, the Lennard Jones potential is a 6 – 12 potential, but its main characteristics are given by the effective potential energy of the H atom:

$$V_{eff} = -\frac{e^2}{4\pi\epsilon_0 r} + \frac{l(l+1)\hbar^2}{2mr^2} \quad -(1)$$

Its gravitational counterpart is:

$$V = -\frac{kx^2}{r} + \frac{(x^2-1)k\alpha}{2r^2} \quad -(2)$$

where

$$k = \frac{L^2}{2m} \quad -(3)$$

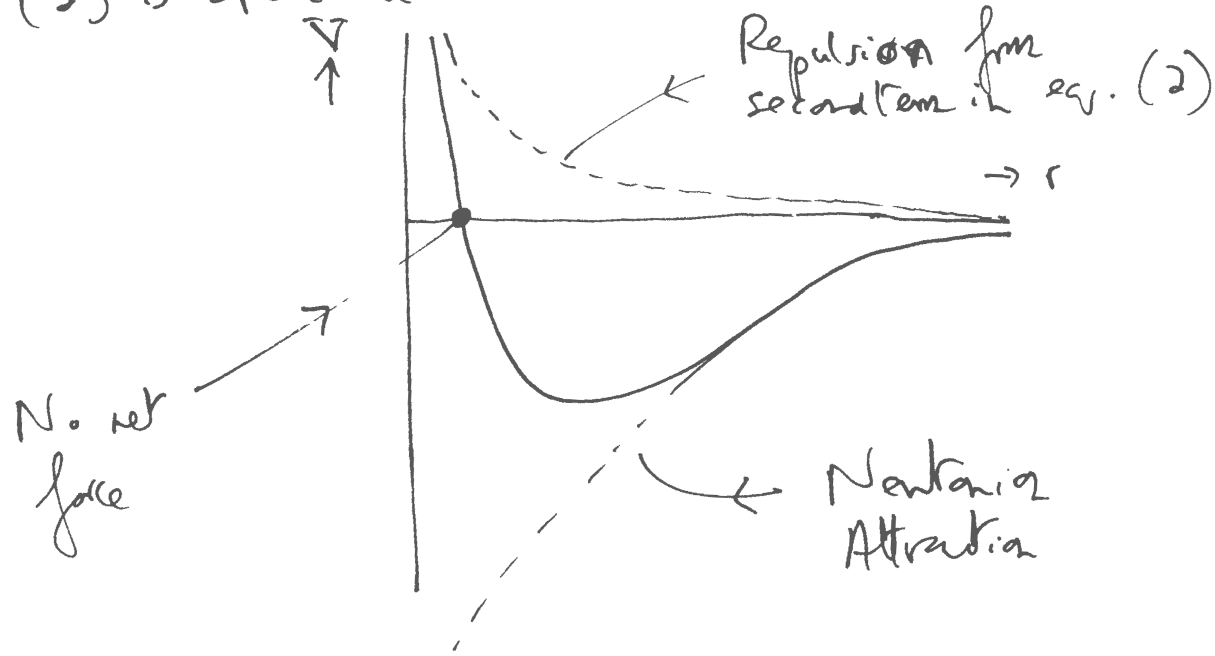
The Newtonian potential is:

$$V_N = -\frac{k}{r} \quad -(4)$$

For

$$x > 1 \quad -(5)$$

eq. (2) is sketched below:



d) For
$$x < 1 \quad -(6)$$
the same result is obtained because V in eq. (2) depends on x^2.

The force is:
$$F = -\frac{\partial V}{\partial x} \quad -(6)$$
$$= -\frac{kx^2}{r^2} + (x^2-1)\frac{\alpha k}{r^3}$$

The condition for a gravitational Lennard-Jones potential is:
$$\boxed{x^2 > 1} \quad -(7)$$

If a stable orbit can be defined by:
$$V = 0 \quad -(8)$$
then
$$\frac{kx^2}{r_0} = \frac{(x^2-1)k\alpha}{2r_0^2} \quad -(9)$$
$$x^2 = \frac{(x^2-1)\alpha}{2r_0} \quad -(10)$$
$$x^2(2r_0 - \alpha) = -\alpha \quad -(11)$$
$$\boxed{x^2 = \frac{\alpha}{\alpha - 2r_0}} \quad -(12)$$