216(2): Newtonian Theory of Light Deflection.

As in note 216(1) eq. (6):

$$1 + \epsilon = \frac{R_0 c^2}{mG} \quad -(1)$$

i.e.

$$\epsilon = \frac{R_0 c^2}{mG} \quad -(2)$$

to an excellent approximation. For the hyperbola:

$$\epsilon = \frac{1}{\sin\theta} \quad -(3)$$

so

$$\sin\theta = \frac{mG}{R_0 c^2} \quad -(4)$$

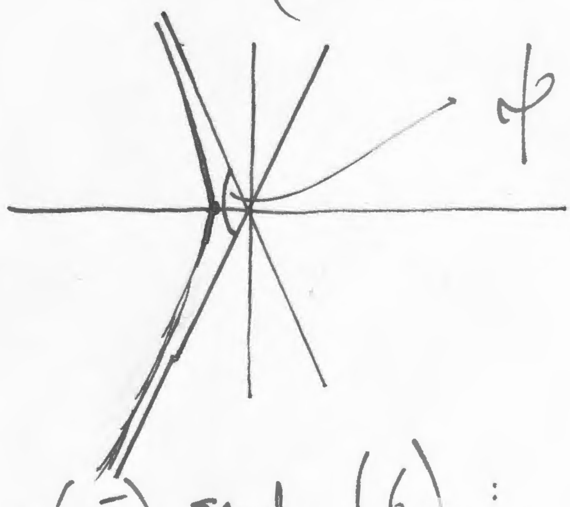
$\sim \theta$

The total deflection is given by the angle between two asymptotes of a hyperbola:

$$\phi = 2\tan^{-1}\frac{b}{a} \quad -(5)$$

where

$$\frac{b}{a} = \frac{1}{(\epsilon^2 - 1)^{1/2}} \quad -(6)$$



From eqs. (5) and (6):

2)

$$\frac{b}{a} = \frac{1}{\epsilon} \quad -(7)$$

for

$$\epsilon \gg 1 \quad -(8)$$

so

$$\phi = 2\tan^{-1}\frac{1}{\epsilon} \sim \frac{2}{\epsilon} \quad -(9)$$

using:

$$\tan^{-1}x = x - \frac{x^3}{3} + \frac{x^5}{5} + \dots \quad -(10)$$

so

$$\boxed{\phi = \frac{2mG}{R_0 c^2} = 2\theta} \quad -(11)$$

In this Newtonian theory:

$$E \rightarrow \frac{1}{2}mc^2, \quad L \rightarrow mcR_0 \quad -(12)$$

and $U(r)$ is neglected. The closest approach is defined by:

$$\cos\theta = 0 \quad -(13)$$

and

$$r = R_0 \quad -(14)$$

so

$$R_0 = \frac{\alpha}{1+\epsilon} \quad -(15)$$

It follows that:

$$\epsilon = \left(1 + \frac{2EL^2}{m^3 M^2 G^2}\right)^{1/2} \quad -(16)$$

$$= \left(1 + x^2\right)^{1/2}$$

where

$$x = \frac{R_0 c}{MG} \quad -(17)$$

So:

$$\epsilon = 1 + x \quad \text{or} \quad 1 - x \quad -(18)$$

The first root gives eq. (6) of note 216(1), QED.

This is the usual Newtonian theory, in which the orbital angular momentum is:

$$L_{orb} = mcR_0 \quad -(19)$$

So the photon mass equation (30) of note 216(1) becomes:

$$m^3 = 8.697 \times 10^{-69} \left(\hbar + L_{orb} \right)^2 \quad -(20)$$

i.e.

$$\boxed{m^3 = 8.697 \times 10^{-69} \left(\hbar + mcR_0 \right)^2} \quad -(21)$$

which can be solved by computer algebra. Here:

$$R_0 = 6.955 \times 10^8 \, m \quad -(22)$$

$$c = 2.998 \times 10^8 \, m \, s^{-1} \quad -(23)$$

If:

$$mcR_0 \gg \hbar \quad -(24)$$

then

$$m = 8.697 \times 10^{-69} \, c^2 R_0^2$$

$$\boxed{m = 3.78 \times 10^{-34} \, \text{kgm}}$$