208(5): Arc Length Along the Hyperbolic Helix Spiral

This was checked using Maxima by Dr. Hart Echardt and gave the result:

$$r = \int_{r_1}^{r_2} \left(1 + \frac{r_0^2}{r^2}\right)^{1/2} dr \quad -(1)$$

$$= \left(r_2^2 + r_0^2\right)^{1/2} - \left(r_1^2 + r_0^2\right)^{1/2}$$

$$+ \frac{r_0}{2}\left(\log_e\left(\left|\left(r_1^2 + r_0^2\right)^{1/2} + r_0\right|\right) + \log_e\left(\left|\left(r_2^2 + r_0^2\right)^{1/2} - r_0\right|\right)\right.$$

$$\left. - \log_e\left(\left|\left(r_2^2 + r_0^2\right)^{1/2} + r_0\right|\right) - \log_e\left(\left|\left(r_1^2 + r_0^2\right)^{1/2} - r_0\right|\right)\right)$$

This result is different from that given in a site from which eq. (5) of note 208(3) was taken. However this does not affect the equation of motion.

$$\text{As} \quad r_2 \to \infty, \quad r_1 \to 0 \quad -(2)$$

then eq. (1) becomes:

$$r \to r_2 - r_0$$

$$+ \frac{r_0}{2}\left(\log_e\left(\left| r_0 + r_0\right|\right) + \log_e\left(\left| r_2 - r_0\right|\right)\right.$$

$$\left. - \log_e\left(\left| r_2 + r_0\right|\right) - \log_e\left(\left| r_0 - r_0\right|\right)\right)$$

$$-(3)$$

2)

i.e. $r \to \infty$ — (4)

This error in the site of standard integrals does not affect the conclusion of rule 208(3). The Maxima code has been tested in many ways by many scientists and is preferred over the incorrect standard integral site.

For such a simple curve:

$$r = \frac{r_0}{\theta} \quad \text{— (5)}$$

the arc length along it is a very complicated expression (1). For other spirals this will probably not be the case.
