1) 186(8): Complete Results for Gravitational Metric

These are summarized in Table 1:

Connection	Torsion	Curvature
$\Gamma^0_{10} = \frac{r_0}{2r(r-r_0)}$	$T^0_{10} = \frac{r_0}{2r(r-r_0)}$	$R^0{}_{011} = 0$
$\Gamma^1_{11} = -\frac{r_0}{2r(r-r_0)}$	0	0
$\Gamma^2_{12} = \frac{1}{r}$	$T^2_{12} = \frac{2}{r}$	$R^2{}_{211} = 0$
$\Gamma^3_{13} = \frac{1}{r}$	$T^3_{13} = \frac{2}{r}$	$R^3{}_{311} = 0$
$\Gamma^3_{23} = \tan\phi$	$T^3_{23} = \tan\phi$	$R^3{}_{322} = 0$

The metric used is:

$$g_{\mu\nu} = \begin{bmatrix} 1-\frac{r_0}{r} & 0 & 0 & 0 \\ 0 & \left(1-\frac{r_0}{r}\right)^{-1} & 0 & 0 \\ 0 & 0 & -r^2 & 0 \\ 0 & 0 & 0 & -r^2\sin^2\phi \end{bmatrix} \quad -(1)$$

with the metric compatibility equation:

$$D_\rho g_{\mu\nu} = 0 \quad -(2)$$

2)

i.e.

$$\partial_\rho g_{\mu\nu} - \Gamma^{\lambda}_{\rho\mu} g_{\lambda\nu} - \Gamma^{\lambda}_{\rho\nu} g_{\mu\lambda} = 0 \quad -(3)$$

In the case of the metric Γ^{1}_{11} it is known that:

$$[D_\mu, D_\nu] V^\rho = -\left(\Gamma^{\lambda}_{\mu\nu} - \Gamma^{\lambda}_{\nu\mu}\right) D_\lambda V^\rho + R^{\rho}{}_{\sigma\mu\nu} V^\sigma \quad -(4)$$

Here:

$$R^{\rho}{}_{\sigma\mu\nu} = \partial_\mu \Gamma^{\rho}_{\nu\sigma} - \partial_\nu \Gamma^{\rho}_{\mu\sigma} + \Gamma^{\rho}_{\mu\lambda}\Gamma^{\lambda}_{\nu\sigma} - \Gamma^{\rho}_{\nu\lambda}\Gamma^{\lambda}_{\mu\sigma} \quad -(5)$$

So for Γ^{1}_{11}: $\quad \mu = \nu = 1 \quad -(6)$

and $\quad [D_1, D_1] V^\rho = R^{\rho}{}_{\sigma 11} V^\sigma = 0 \quad -(7)$

i.e. $\quad T^{\lambda}{}_{11} = 0 \quad -(8)$

$\quad R^{\rho}{}_{\sigma 11} = 0, \quad -(9)$

and both the torsion and curvature vanish.

The other curvatures of Table 1 are worked out using eq. (5) and all curvatures vanish.

Conclusion

For a metric of type (1), straightforward use of eq. (3) gives five non-zero connections. The connection relevant to gravitation and theory is Γ^{0}_{10}, i.e.

$$\Gamma^{0}_{10} = -\Gamma^{0}_{01}. \quad -(10)$$