1) 181(2): Hamilton Jacobi Equation from Fermion Equation.

Define the ECE wave equation as:

$$(\square + R_0 + R_1)\phi = 0 \quad -(1)$$

where

$$R = R_0 + R_1, \quad -(2)$$

and

$$R_0 = \left(\frac{m_0 c}{\hbar}\right)^2, \quad -(3)$$

$$R_1 = \kappa^\mu \kappa_\mu = \frac{\omega^2}{c^2} - \kappa^2. \quad -(4)$$

The advantage of this format is that it is based on a constant hamiltonian:

$$H = \frac{1}{2} m_0 c^2 \quad -(5)$$

The interaction of a field / particle with the particle of observed mass m_0 is described by R_1.

In the absence of interaction:

$$\left(\square + \left(\frac{m_0 c}{\hbar}\right)^2\right)\phi = 0 \quad -(6)$$

whose classical description is:

$$p^\mu p_\mu = m_0^2 c^2. \quad -(7)$$

Eq. (7) can be factorized into the free fermion equations:

$$(E - c\underline{\sigma}\cdot\underline{p})\phi^R = m_0 c^2 \phi^L \quad -(8)$$

$$(E + c\underline{\sigma}\cdot\underline{p})\phi^L = m_0 c^2 \phi^R \quad -(9)$$

2) In the position representation $\underline{p}$ is the vector valued relativistic momentum:

$$\underline{p} = \gamma m \underline{v} \quad - (10)$$

and E is the total relativistic energy:

$$E = \gamma m_0 c^2 \quad - (11)$$

Multiply (8) by (9) to obtain:

$$E^2 - c^2 p^2 = m_0^2 c^4 \quad - (12)$$

which is eq. (7).

The interaction is described by:

$$p^\mu \rightarrow p^\mu - \hbar \kappa^\mu \quad - (13)$$

where

$$R_1 = \kappa^\mu \kappa_\mu = \frac{\omega^2}{c^2} - \kappa^2 \quad - (14)$$

Here

$$p^\mu = \left(\frac{E}{c}, \underline{p} \right), \quad \kappa^\mu = \left(\frac{\omega}{c}, \underline{\kappa} \right) \quad - (15)$$

so

$$E \rightarrow E - \hbar\omega, \quad \underline{p} \rightarrow \underline{p} - \hbar \underline{\kappa} \quad - (16)$$

Therefore eqs. (8) and (9) become:

$$\left(E - \hbar\omega - c\underline{\sigma} \cdot (\underline{p} - \hbar\underline{\kappa}) \right) \phi^R = m_0 c^2 \phi^L \quad - (17)$$

$$\left(E - \hbar\omega + c\underline{\sigma} \cdot (\underline{p} - \hbar\underline{\kappa}) \right) \phi^L = m_0 c^2 \phi^R \quad - (18)$$

Adding eqs. (17) and (18) to obtain

3)

From eq. (18):

$$\phi^R = \frac{1}{m_0 c^2}\left(E - \hbar\omega + c\underline{\sigma}\cdot(\underline{p} - \hbar\underline{\kappa})\right)\phi^L \quad -(19)$$

Therefore eq. (17) becomes:

$$\left(E - \hbar\omega - c\underline{\sigma}\cdot(\underline{p} - \hbar\underline{\kappa})\right)\left(\left(E - \hbar\omega + c\underline{\sigma}\cdot(\underline{p} - \hbar\underline{\kappa}\right)\phi^L\right) = m_0^2 c^4 \phi^L$$
$$-(20)$$

If eqs. (17) and (18) are multiplied together, the result is

$$(E - \hbar\omega)^2 - c^2(\underline{p} - \hbar\underline{\kappa})\cdot(\underline{p} - \hbar\underline{\kappa}) = m_0^2 c^4 \quad -(21)$$

which is the relativistic Hamilton Jacobi equation:

$$\boxed{(p^\mu - \hbar\kappa^\mu)(p_\mu - \hbar\kappa_\mu) = m_0^2 c^2} \quad -(22)$$

We wish to incorporate R theory into eq. (22). This is done by expanding the left hand side of the equation:

$$p^\mu p_\mu + \hbar^2 \kappa^\mu \kappa_\mu - \hbar(\kappa^\mu p_\mu + p^\mu \kappa_\mu) = m_0^2 c^2$$
$$-(23)$$

Write the left hand side as:

$$p^\mu p_\mu - \hbar^2 R_1 = m_0^2 c^2 \quad -(24)$$

where R_1 is defined by eq. (1). Eq. (24) is the classical equivalent of eq. (1) using:

4)

$$p^\mu = i\hbar\, \partial^\mu \quad -(25)$$

so

$$p^\mu p_\mu = -\hbar^2 \partial^\mu \partial_\mu = -\hbar^2 \Box \quad -(26)$$

Therefore:

$$\hbar^2 R_1 = -\hbar^2 \kappa^\mu \kappa_\mu + \hbar(\kappa^\mu p_\mu + p^\mu \kappa_\mu) \quad -(27)$$

Finally use:

$$p^\mu \rightarrow p^\mu + \hbar \kappa^\mu \quad -(28)$$

which is the minimal prescription for describing the interaction of the free particle with another field / particle. Therefore in eq. (27)

$$p_\mu = \hbar \kappa_\mu \quad -(29)$$

and

$$R_1 = \kappa^\mu \kappa_\mu \quad -(30)$$

which is eq. (4), QED.

It has been shown that eq. (1) is the quantized version of the Hamilton Jacobi equation (22), given the minimal prescription (28). Particle collisions can be described by:

$$R = R_0 + R_1 \quad -(31)$$

$$= \left(\frac{m_0 c}{\hbar}\right)^2 + \kappa^\mu \kappa_\mu$$

5)

i.e.

$$\boxed{R = \left(\frac{m_0 c}{\hbar}\right)^2 + \frac{\omega^2}{c^2} - \kappa^2} \quad -(32)$$

Here ω and κ are the angular frequency and wavenumber associated with the matter wave interacting with the free particle of measured mass m_0.

The wave equation for the interaction is:

$$\boxed{\left(\Box + \frac{\omega^2}{c^2} - \kappa^2 + \left(\frac{m_0 c}{\hbar}\right)^2\right)\phi = 0} \quad -(33)$$

The correct description of particle interaction on the classical level is therefore:

$$p^\mu p_\mu = \hbar^2 \kappa^\mu \kappa_\mu + m_0^2 c^2 \quad -(34)$$

i.e.

$$\frac{p^\mu p_\mu}{\hbar^2} = R_1 + R_0 \quad -(35)$$

or

$$\boxed{p^\mu p_\mu = \hbar^2 (R_1 + R_0)} \quad -(36)$$

Eq. (36) may now be applied to particle collisions.