175(6): Evaluation with Particle in a Box Wave functions

The problem is to evaluate the expectation value of the right hand side of the equation:

$$[\hat{x}^2, \hat{p}^2]\psi - 2\hbar^2\psi + 4i\hbar\hat{x}\hat{p}\psi \quad -(1)$$

with the particle in a box wave function:

$$\psi_n = \left(\frac{2}{L}\right)^{1/2} \sin\left(\frac{n\pi x}{L}\right) \quad -(2)$$

$$n = 1, 2, 3, \ldots$$

Therefore:

$$\langle \hat{x}\hat{p} \rangle = -\frac{2i\hbar}{L}\int_0^L \sin\left(\frac{n\pi x}{L}\right) x \frac{d}{dx} \sin\left(\frac{n\pi x}{L}\right) dx \quad -(3)$$

$$= -\frac{2i\hbar n\pi}{L^2}\int_0^L x \sin\left(\frac{n\pi x}{L}\right) \cos\left(\frac{n\pi x}{L}\right) dx$$

I cannot find this in tables of integrals but it is imaginary valued and so will contribute to the expectation value of the right hand side of eq. (1).

For the two dimensional square well:

$$\psi_{n_1 n_2}(x, y) = \left(\frac{4}{L_1 L_2}\right)^{1/2} \sin\left(\frac{n_1 \pi x}{L_1}\right) \sin\left(\frac{n_2 \pi y}{L_2}\right) \quad -(4)$$

and:

$$\hat{p} = -i\hbar\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y}\right) \quad -(5)$$