1) 173(3): The g Factor and Spin-Orbit Coupling from the Fermion Equation.

Write the fermion equation in the format:

$$\left((E - e\phi) + c\,\underline{\sigma}\cdot\underline{\pi}\right)\phi^L = mc^2\phi^R \quad -(1)$$

$$\left((E - e\phi) - c\,\underline{\sigma}\cdot\underline{\pi}\right)\phi^R = mc^2\phi^L \quad -(2)$$

Note carefully that the eigenvalue mc^2 is positive.
The concept of "negative energy" is nowhere used.

From eq. (2):

$$\phi^R = \left(\frac{mc^2}{E - c\,\underline{\sigma}\cdot\underline{\pi} - e\phi}\right)\phi^L \quad -(3)$$

where

$$\underline{\pi} = \underline{p} - e\underline{A} \quad -(4)$$

so

$$\left((E - e\phi)^2 - c^2\,\underline{\sigma}\cdot\underline{\pi}\,\underline{\sigma}\cdot\underline{\pi}\right)\phi^L = (mc^2)^2\phi^L \quad -(5)$$

and

$$\left(E - e\phi - c^2\frac{\underline{\sigma}\cdot\underline{\pi}\,\underline{\sigma}\cdot\underline{\pi}}{E - e\phi}\right)\phi^L = \left(\frac{m^2c^4}{E - e\phi}\right)\phi^L \quad -(6)$$

Add mc^2 to both sides:

$$\left(E + mc^2 - e\phi - \frac{c^2\,\underline{\sigma}\cdot\underline{\pi}\,\underline{\sigma}\cdot\underline{\pi}}{E - e\phi}\right)\phi^L = \left(\left(\frac{m^2c^4}{E - e\phi}\right) + mc^2\right)\phi^L \quad -(7)$$

In the non-relativistic approximation:

2)

$$\left(2mc^2 - e\phi - c^2\frac{\underline{\sigma}\cdot\underline{\Pi}\ \underline{\sigma}\cdot\underline{\Pi}}{E-e\phi}\right)\phi^L = \left(\frac{m^2c^4}{E-e\phi} + mc^2\right)\phi^L \quad -(8)$$

i.e

$$(2mc^2 - e\phi)(E-e\phi)\phi^L = \left(c^2\,\underline{\sigma}\cdot\underline{\Pi}\ \underline{\sigma}\cdot\underline{\Pi} + mc^2\left(mc^2 + E - e\phi\right)\right)\phi^L$$

$$= \left(c^2\,\underline{\sigma}\cdot\underline{\Pi}\ \underline{\sigma}\cdot\underline{\Pi} + mc^2\left(2mc^2 - e\phi\right)\right)\phi^L \quad -(9)$$

so:

$$\left(E - e\phi\right)\phi^L = \left(\left(\frac{c^2\,\underline{\sigma}\cdot\underline{\Pi}\ \underline{\sigma}\cdot\underline{\Pi}}{2mc^2 - e\phi}\right) + mc^2\right)\phi^L \quad -(10)$$

i.e

$$E\phi^L = H\phi^L \quad -(11)$$

where

$$H = mc^2 + e\phi + \frac{c^2\,\underline{\sigma}\cdot\underline{\Pi}\ \underline{\sigma}\cdot\underline{\Pi}}{2mc^2 - e\phi} \quad -(11)$$

$$= mc^2 + e\phi + \frac{1}{2m}\,\underline{\sigma}\cdot\underline{\Pi}\left(1 - \frac{e\phi}{2mc^2}\right)\underline{\sigma}\cdot\underline{\Pi} \quad -(12)$$

which is the same result as the standard representation but without negative energy.