165(7): R Theory of Group Velocity v_g

The de Broglie equation:

$$v_g v_p = c^2 \quad -(1)$$

is valid only in special relativity. In general relativity, as shown in note 165(5):

$$v_g = c^2 \frac{\kappa}{\omega} + \frac{1}{2} \frac{\hbar c}{\gamma R^{1/2}} \frac{\partial R}{\partial p} \quad -(2)$$

First simplify eq. (2) using:

$$\gamma = \frac{\hbar \omega}{mc^2} = \frac{\omega}{cR^{1/2}} \quad -(3)$$

so

$$v_g = \frac{c^2}{\omega}\left(\kappa + \frac{1}{2} \hbar \frac{\partial R}{\partial p}\right) \quad -(4)$$

Now use:

$$p = \hbar \kappa \quad -(5)$$

so:

$$\boxed{v_g = \frac{c^2}{\omega}\left(\kappa + \frac{1}{2} \frac{\partial R}{\partial \kappa}\right)} \quad -(5)$$

From note 165(6):

$$\boxed{v_g = c\left(1 - \left(\frac{c}{\omega}\right)^2 R\right)^{1/2}} \quad -(6)$$

The phase velocity is:

$$v_p = \frac{\omega}{\kappa} \quad -(7)$$

So in R theory and general relativity, the group velocity is defined by eqns (5) and (6).

2)

Recent observations have shown (from 2006 onwards) that v_g may be zero or negative, or greater than c. Eqns. (5) and (6) describe these possibilities straightforwardly as follows.

1) <u>Zero v_g</u>

In this case:
$$R = \left(\frac{\omega}{c}\right)^2. \quad -(8)$$

2) <u>Negative v_g</u>

In this case $\partial R / \partial \kappa$ is negative and greater than κ, in eqn (5). In eqn. (6):
$$v_g = -c\left(1 - \left(\frac{c}{\omega}\right)^2 R\right)^{1/2} \quad -(9)$$

i.e the negative root of:
$$v_g^2 = c^2\left(1 - \left(\frac{c}{\omega}\right)^2 R\right). \quad -(10)$$

3) <u>$v_g > c$</u>

In this case R is negative in eq. (6).

Therefore the de Broglie eq. (1) is generalized to:
$$\boxed{\begin{aligned} v_g v_p &= c^2\left(1 + \frac{1}{2}\frac{1}{\kappa}\frac{\partial R}{\partial \kappa}\right) \quad -(11) \\ &= \omega\frac{c}{\kappa}\left(1 - \left(\frac{c}{\omega}\right)^2 R\right)^{1/2} \quad -(12) \end{aligned}}$$