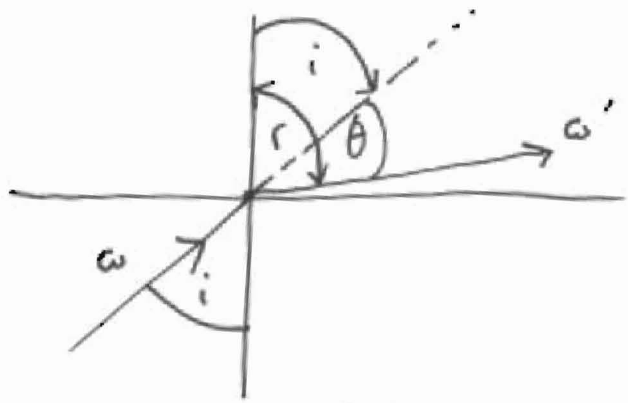
165(4): Unified Theory of Refraction and Scattering.

From the optical theory of refraction, note 165(3) showed that:

$$\frac{v'}{v} = \frac{\sin i}{\sin r} \quad -(1)$$

with reference to figure (1)

Fig. (1)



with:

$$\theta = r - i \quad -(2)$$

In scattering theory:

$$\omega_0 = c R_2^{1/2} = \frac{1}{\omega - \omega'} \left[\omega\omega' - \left(c^2 R_1 + \left(\omega^2 - c^2 R_1\right)^{1/2} \left(\omega'^2 - c^2 R_1\right)^{1/2} \cos\theta \right) \right] \quad -(3)$$

So:

$$R_1 = \frac{1}{2ac^2} \left(-b \pm \left(b^2 - 4ac' \right)^{1/2} \right) \quad -(4)$$

$$a = 1 - \cos^2\theta$$
$$b = \left(\omega'^2 + \omega^2\right)\cos^2\theta - 2A$$
$$c' = A^2 - \omega^2\omega'^2 \cos^2\theta$$
$$A = \omega\omega' - (\omega - \omega')\,\omega_0$$

From the optical theory, eq. (20) of note 165(3):

2)

$$\left(\frac{\sin i}{\sin r}\right)^2 = \left(\frac{\omega}{\omega'}\right)^2 \left[\frac{\omega'^2 - c^2 R_1}{\omega^2 - c^2 R_1}\right] \quad -(5)$$

where R_1 is given by eq. (4). From eq. (5):

$$\boxed{\begin{aligned} R_1 &= \left(\frac{\omega'}{c}\right)^2 \left[\frac{1 - (\sin i / \sin r)^2}{1 - \left(\frac{\omega'}{\omega}\right)^2 \left(\frac{\sin i}{\sin r}\right)^2}\right] \quad -(6) \\ &= \frac{1}{2ac^2}\left(-b \pm (b^2 - 4ac')^{1/2}\right), \\ \theta &= r - i \end{aligned}}$$

Eq. (6) unifies the macroscopic theory of refraction with the particle theory of refraction. The effective R_1 and R_2 may be found from eq. (6).

Errata

1) In eq. (43) of UFT 160 and eq. (36) of UFT 161:

$$A = \omega\omega' - x_2(\omega - \omega').$$

2) Eq. (41) of UFT 161 should be:

$$\omega\omega' - x_2(\omega - \omega') = \omega\omega' \cos\theta$$