163(7): Interpretation of Mass in Elastic Scattering

As in note 163(4), elastic scattering is developed by considering the special case:

$$\gamma m c^2 + m_2 c^2 = \gamma' m c^2 + \gamma'' m_2 c^2, \quad -(1)$$

$$\gamma = \gamma', \quad -(2)$$

$$\omega = \omega'. \quad -(3)$$

and

$$\kappa''^2 = \kappa^2 + \kappa'^2 - 2\kappa\kappa'\cos\theta, \quad -(4)$$

$$\kappa^2 = \kappa'^2. \quad -(5)$$

These equations give the result:

$$m = \frac{\hbar\omega}{c^2}. \quad -(6)$$

It is obvious that $m$ is different from the rest mass, defined by

$$\gamma m_0 = \frac{\hbar\omega}{c^2}. \quad -(7)$$

From eqs. (6) and (7):

$$\boxed{m = \gamma m_0} \quad -(8)$$

Eq. (8) means that the "dynamic mass" $m$ in the collision process is $\gamma m_0$ where $m_0$ is the mass defined by the rest frequency:

$$m_0 c^2 = \hbar\omega_0. \quad -(9)$$

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Therefore:

$$\frac{R}{R_0} = \left(\frac{m}{m_0}\right)^2 = \gamma^2 \quad -(10)$$

If the velocity of the particle is zero:

$$m = m_0. \quad -(11)$$

If the velocity of the particle approaches c:

$$m \to \infty. \quad -(12)$$

If it is possible to consider a hypothetically defined:

$$m_0 \to 0 \quad -(13)$$

then m is defined by the hyperrelativistic limit and is indeterminate:

$$m \to \frac{0}{0}. \quad -(14)$$

From eq. (14), m can remain infinitesimally close to zero. In this case eq. (1) becomes:

$$\hbar\omega + m_2 c^2 = \hbar\omega' + \gamma'' m_2 c^2 \quad -(15)$$

which is the equation of the Compton effect, in which

$$\omega \neq \omega'. \quad -(16)$$

It seems that the covariant mass ratio:

$$\boxed{\frac{m}{m_0} = \gamma} \quad -(17)$$

is self-consistent if $m_2$ is assumed constant.