1)

162(2): Expression for x_2 in terms of x_1

Start with eq. (20) of note 162(2), then:

$$x_1^2 = 2x_2^2 - (\omega'^2 + \omega''^2) - (\omega'' - \omega')^2 + 2(\omega''^2 - x_2^2)^{1/2}(\omega'^2 - x_2^2)^{1/2}\cos\theta \quad -(1)$$

Let
$$x := x_2^2. \quad -(2)$$

then:
$$x + (\omega''^2 - x)^{1/2}(\omega'^2 - x)^{1/2}\cos\theta = A \quad -(3)$$

where:
$$A = \frac{1}{2}\left(x_1^2 + (\omega''^2 + \omega'^2) + (\omega'' - \omega')^2\right) \quad -(4)$$

so
$$(\omega''^2 - x)(\omega'^2 - x)\cos^2\theta = (A - x)^2, \quad -(5)$$

$$\left(\omega''^2\omega'^2 - x(\omega'^2 + \omega''^2) + x^2\right)\cos^2\theta = A^2 - 2Ax + x^2,$$

i.e.
$$x^2(1 - \cos^2\theta) + \left((\omega'^2 + \omega''^2)\cos^2\theta - 2A\right)x + A^2 - \omega''^2\omega'^2\cos^2\theta = 0 \quad -(6)$$

which is:
$$ax^2 + bx + c' = 0, \quad -(7)$$

where
$$a = 1 - \cos^2\theta,$$
$$b = (\omega''^2 + \omega'^2)\cos^2\theta - 2A,$$
$$c' = A^2 - \omega''^2\omega'^2\cos^2\theta$$

so
$$x_1^2 = \frac{1}{2a}\left(-b \pm (b^2 - 4ac)^{1/2}\right) \quad -(8)$$