3. Numerical integration of Einstein's original integral (39)

Einstein's formula (39) for light deflection depends on the radius parameters R_0 and r_0. R_0 represents the radius of the sun ($6.955*10^8$ m) while r_0, sometimes called the "Schwarzschild radius", is only 2954 m. Therefore we have

$$r_0 << R_0$$

which implies according to Eq. (37) that

$$b_0 \approx R_0.$$

The integral (39)

$$\Delta\phi = 2\int_0^{1/R_0}\left(\frac{R_0 - r_0}{R_0{}^3} - u^2 + r_0\, u^3\right)^{-1/2} du \qquad (47)$$

is not solvable analytically and needs to be evaluated numerically. First it is to be noted that the square root in the integrand has zero crossings, leading to infinite values of the integrand. The argument of the square root

$$A(u) = \frac{R_0 - r_0}{R_0{}^3} - u^2 + r_0\, u^3 \qquad (48)$$

Is plotted in Fig. 1 where u is the inverse radius parameter

$$u = \frac{1}{r} \qquad (49)$$

and the relevant range for integral (47) is 0 to $1.4378*10^{-9}$ m^{-1}. Numerical analysis shows that there is a zero crossing exactly at this value so that the argument A(u) is positive in the definition range of the integral. The integrand of (47) itself is graphed in Fig. 2. It has a sharp pole at $u=1/R_0$. The numerical result is

$$\Delta\phi = 3.1416$$

which is by six orders of magnitude larger than Einstein's result of

$$\Delta\phi_{Einstein} = 8.4955 * 10^{-6}.$$

This discrepancy deserves precise analysis. According to Fig. 2, the value of the integral is mainly determined by the region near to $1/R_0$. Increasing the boundary value R_0 by 10% leads to a decrease of $\Delta\phi$ to 2.28. This may give a hint to the sensitivity of the result on the integration boundary. There is no change in orders of magnitude. The numerical accuracy of the integration was reported to be 10^{-12}, much lower than the range of both results. The calculation was performed by the computer algebra system Maxima and was checked by evaluating the integral independently in Mathematica. Both programs yielded the identical result. So the discrepancy to Einstein's calculation cannot be explained by numerical instability of the integral value.

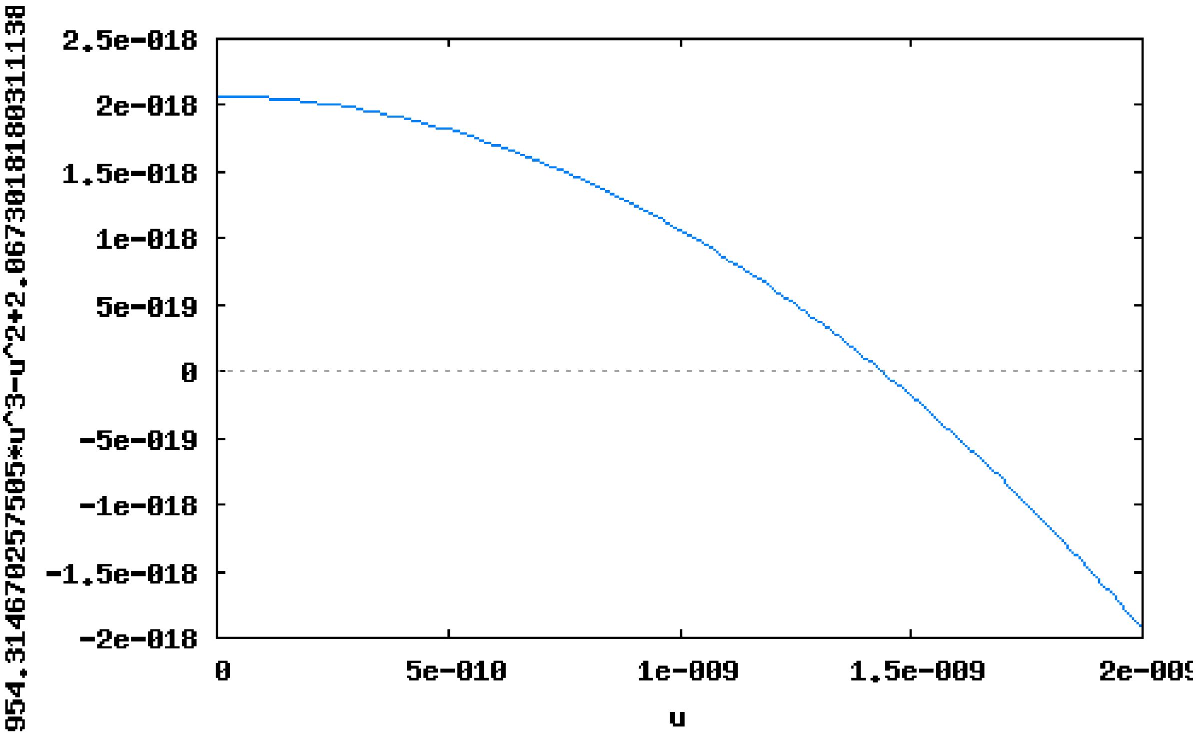

Fig. 1. u dependence of square root argument in Eq. (47).

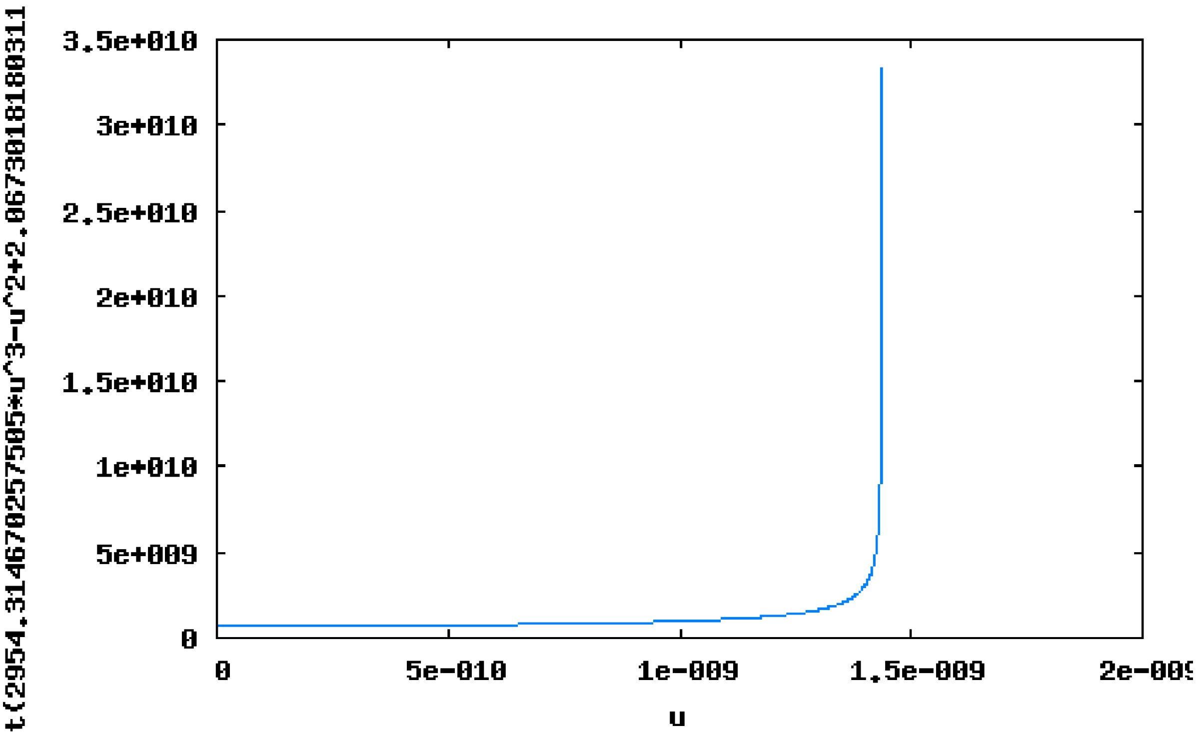

Fig. 2. u dependence of integrand in Eq. (47).

b_0	R_0	$\Delta\phi$
$6.95501*10^{8}$	$6.955*10^{8}$	3.1416
$1*10^{3}$	$6.955*10^{8}$	$2.8756*10^{-6}$
$6.95501*10^{8}$	$6.955*10^{14}$	$2.0000*10^{-6}$

Table 1. Variation of parameters in integral (47).

In order to see the impact of the parameters b_0 and R_0 on the result we have changed both parameters separately as shown in Table 1. It is required to reduce b_0 by five orders of magnitude to have $\Delta\phi$ covering the experimentally observed range. Alternatively, R_0 has to be increased by six orders of magnitudes. Thus Einstein's result is not consistent in any way.

4. Numerical integration of the correct integral (16) and estimation of photon mass

The correct formula for the light deflection is (Eq. (16))

$$\Delta\phi = 2\int_0^{1/R_0}\left(\frac{1}{b^2} - (1 - r_0 u)\left(\frac{1}{a^2} + u^2\right)\right)^{-1/2} du \qquad (50)$$

with a and b being parameters having to be determined in such a way that the experimental result for $\Delta\phi$ is obtained. From Eq. (12) we have

$$a = \frac{L}{mc},\ \ b = \frac{cL}{E} \qquad (51)$$

where m is the photon mass and E the photon energy

$$E = \hbar\omega. \qquad (52)$$

From Eq. (51) follows

$$a = \frac{\hbar\omega}{mc^2} b. \qquad (53)$$

In the first approximation we have for the orbital angular momentum of the photon

$$L = mr^2\frac{d\phi}{dt} = mr^2\omega_L \qquad (54)$$

with

$$\omega_L = \frac{v}{r}. \qquad (55)$$

If the photon is travelling close to c, it is

$$\omega_L \approx \frac{c}{r}, \qquad (56)$$

so

$$L = mrc \qquad (57)$$

and

$$a = r. \qquad (58)$$

Assuming

$$a = R_0 \qquad (59)$$

we have from (53)

$$m = \frac{\hbar\omega}{c^2 R_0} b \qquad (60)$$

as an estimation for the photon mass. The reduced Planck constant in SI units is

$$\hbar = 1.05457 * 10^{-34} \text{ Js} \qquad (61)$$

and the average frequency of visible light is chosen to be

$$\omega = 1.0 * 10^{16} \text{ /s.} \qquad (62)$$

The parameter b is to be determined in such a way that integral (50) yields the experimental value of $\Delta\phi$. Numerical analysis shows that b= R_0 gives negative values of the square root argument of (50). Therefore b must be chosen much smaller. Interestingly, the choice of

$$b = r_0 \qquad (63)$$

gives

$$\Delta\phi = 8.4955 * 10^{-6} \qquad (64)$$

which is close to the experimental value of $8.484 * 10^{-6}$, see Eq. (41). This astonishing result shows that b obviously has a physical meaning. The square root argument in the integral (50) is shown in Fig. 3. It behaves regular and has only a very weak u dependence in the range of interest. The same holds for the integrand itself (Fig. 4). Therefore the numerical results are reliable. From Eq. (60) we get the estimation of the photon mass:

$$\boxed{m \approx 5 * 10^{-41} \text{ kg.}} \qquad (65)$$

This is the first estimation of the photon mass, the existence of which was predicted by Evans and Vigier [8].

Finally we derive Einstein's result (1) for light deflection in an approximation. As can be seen from Fig. 4, the u variation of the integrand in Eq. (50) is very weak. Therefore we can neglect the total u dependence, leading to

$$\Delta\phi \approx 2\int_0^{1/R_0} b\, du = 2\int_0^{1/R_0} r_0\, du = 2\frac{r_0}{R_0} = \frac{4MG}{c^2}. \qquad (66)$$

This is the correct way for deriving this result, proving again that Einstein's calculation was wrong.

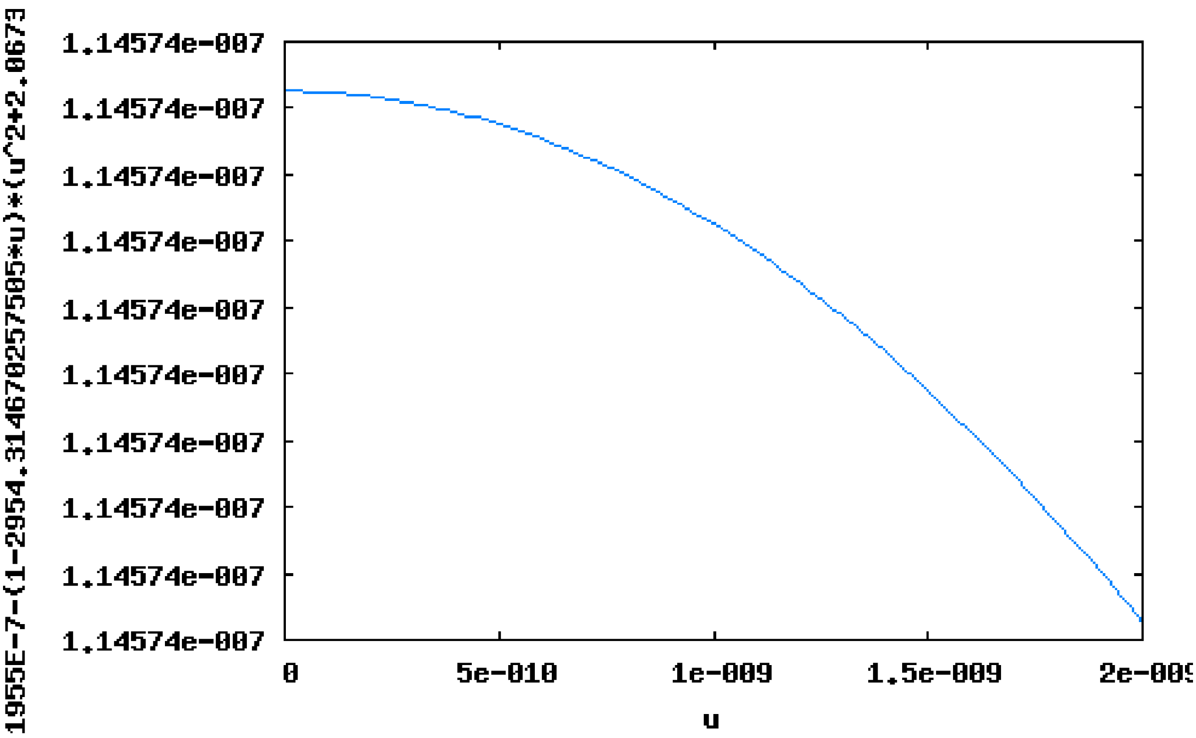

Fig. 3. u dependence of square root argument in Eq. (50).

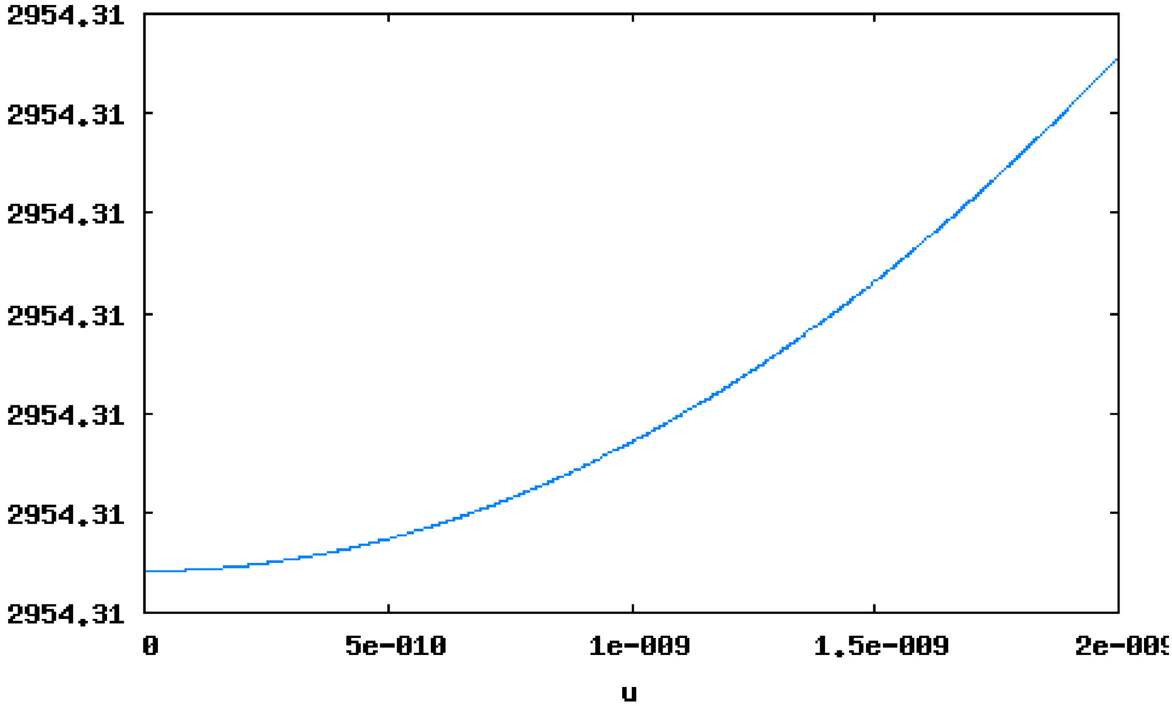

Fig. 4. u dependence of integrand in Eq. (50).