144(7): Interaction of Spin and Orbital $\underline{E}$ and $\underline{B}$

The basic structure of the theory is:

$$A = \frac{m}{e} D \wedge r \quad -(1)$$

$$F = D \wedge A \quad -(2)$$

The two-form on the LHS of eq. (1) is re-arranged to give the one-form potential A^a_μ used in eq. (2). This gives:

$$\underline{A}^a_{orb} = \frac{m}{e}\left(\frac{\partial \underline{r}^a}{\partial t} + c\nabla r^a_0 + c\omega^a_{0b}\underline{r}^b - c r^b_0 \underline{\omega}^a_b\right) \quad -(3)$$

$$\underline{A}^a_{spin} = \frac{m}{e}\,\frac{1}{c}\left(\nabla \times \underline{r}^a - \underline{\omega}^a_b \times \underline{r}^b\right) \quad -(4)$$

It is seen that $\underline{A}^a_{spin}$ is c times smaller than $\underline{A}^a_{orb}$.

Analogously, the magnetic flux density $\underline{B}$ is c times smaller in magnitude than the electric field strength $\underline{E}$. Similarly, the gravitomagnetic field $\underline{\Omega}$ is c times smaller than the $\underline{g}$ gravitational quantity of Newtonian theory. Therefore $\underline{A}^a_{spin}$ is going to be a small correction to the usual (Heaviside) $\underline{A}^a$ orbital. We can write:

2)

$$|\underline{A}^a_{orb}| = c\,|\underline{A}^a_{spin}|. \quad -(5)$$

Analogously:

$$|\underline{E}| = c\,|\underline{B}| \quad -(6)$$

in S.I. units, and:

$$|\underline{g}| = c\,|\underline{\Omega}|. \quad -(7)$$

Now write:

$$\underline{B}^a(tot) = \underline{B}^a_{orb} + \underline{B}^a_{spin} \quad -(8)$$

$$\underline{E}^a(tot) = \underline{E}^a_{orb} + \underline{E}^a_{spin} \quad -(9)$$

Note that:

$$|\underline{E}^a_{orb}| = c\,|\underline{E}^a_{spin}| \quad -(10)$$

$$|\underline{B}^a_{orb}| = c\,|\underline{B}^a_{spin}| \quad -(11)$$

Thus:

$$\underline{\nabla} \cdot \underline{B}^a(tot) = 0 \quad -(12)$$

$$\underline{\nabla} \times \underline{E}^a(tot) + \frac{\partial \underline{B}^a(tot)}{\partial t} = \underline{0} \quad -(13)$$

$$\underline{\nabla} \cdot \underline{E}^a(tot) = \rho^a(tot) / \epsilon_0 \quad -(14)$$

$$\underline{\nabla} \times \underline{B}^a(tot) - \frac{1}{c^2} \frac{\partial \underline{E}^a(tot)}{\partial t} = \mu_0 \underline{J}^a(tot) \quad -(15)$$

The total $\underline{E}^a$ and $\underline{B}^a$ fields in eqns (8) and (9) are dominated by the orbital fields, which have a magnitude c times greater than the spin fields. However, a possible solution of eq. (13) is:

$$\nabla \times \underline{E}^a(\text{orbital}) + \frac{\partial \underline{B}^a}{\partial t}(\text{spin}) = \underline{0} \quad -(16)$$

and another possible solution is:

$$\nabla \times \underline{E}^a(\text{spin}) + \frac{\partial \underline{B}^a}{\partial t}(\text{orbital}) = \underline{0} \quad -(17)$$

In certain circumstances the orbital electric field may induce a spin magnetic field, and vice versa. The other two possible Faraday laws of induction are:

$$\nabla \times \underline{E}^a(\text{orbital}) + \frac{\partial \underline{B}^a}{\partial t}(\text{orbital}) = \underline{0} \quad -(18)$$

$$\nabla \times \underline{E}^a(\text{spin}) + \frac{\partial \underline{B}^a}{\partial t}(\text{spin}) = \underline{0} \quad -(19)$$

The Coulomb Law

This is:

$$\nabla \cdot \underline{E}(\text{tot.}) = \rho(\text{total}) / \epsilon_0 \quad -(20)$$

where

$$\underline{E}(\text{tot}) = \underline{E}(\text{orb}) + \underline{E}(\text{spin}) \quad -(21)$$

4)

Here:

$$\underline{E}(\text{orb}) = -\frac{\partial \underline{A}}{\partial t}(\text{orb}) - c\underline{\nabla} A_0(\text{orb}) - c\,\omega_{0b}\,\underline{A}^b_{\text{orb}} + c A^b_0(\text{orb})\,\underline{\omega}_b \quad -(22)$$

$$\underline{E}(\text{spin}) = -\frac{\partial \underline{A}}{\partial t}(\text{spin}) - c\underline{\nabla} A_0(\text{spin}) - c\,\omega_{0b}\,\underline{A}^b_{\text{spin}} + c A^b_0(\text{spin})\,\underline{\omega}_b. \quad -(23)$$

Here:

$$\underline{\nabla} \cdot \underline{A}(\text{spin}) = 0 \quad -(24)$$

$$\underline{\nabla} \times \underline{A}(\text{orb}) + \frac{\partial \underline{A}}{\partial t}(\text{spin}) = \underline{0} \quad -(25)$$

$$\underline{\nabla} \cdot \underline{A}(\text{orb}) = \xi \quad -(26)$$

$$\underline{\nabla} \times \underline{A}(\text{spin}) - \frac{1}{c^2}\frac{\partial \underline{A}}{\partial t}(\text{orb}) = \underline{\xi} \quad -(27)$$

Here:

$$-(28)$$

$$\underline{A}_{\text{orb}} = \frac{m}{e}\left(\frac{\partial \underline{r}}{\partial t} + c\underline{\nabla} r_0 + c\,\omega_{0b}\,\underline{r}^b - c\,r^b_0\,\underline{\omega}_b\right)$$

$$\frac{\underline{A}_{\text{spin}}}{c} = \frac{m}{e}\left(\underline{\nabla} \times \underline{r} - \underline{\omega}_b \times \underline{r}^b\right) \quad -(29)$$