144(4): Plane Wave Solution for Velocity and Acceleration In the Limit of Vanishing Spin Connection

In this case:

$$\frac{\underline{v}}{c} = \frac{1}{c}\frac{\partial \underline{r}}{\partial t} + \underline{\nabla}\, r_0 \quad -(1)$$

By anti-symmetry:

$$\frac{1}{c}\frac{\partial \underline{r}}{\partial t} = \nabla r_0 \quad -(2)$$

The line function is:

$$\underset{\sim}{r} = \frac{r}{\sqrt{2}}\left(\underline{i} - i\underline{j}\right) e^{i(\omega t - \kappa Z)} \quad -(3)$$

which is a plane wave. The positive four vector is:

$$r^{\mu} = \left(r_0, \underset{\sim}{r}\right) \quad -(4)$$

Therefore:

$$\boxed{\underline{v} = \frac{\partial \underline{r}}{\partial t} = c\nabla r_0 = i\omega \underline{r}} \quad -(5)$$

and

$$\underline{\nabla}\, r_0 = i\frac{\omega}{c}\underset{\sim}{r} = i\kappa \underline{r} \quad -(6)$$

Therefore

$$\boxed{\underline{v} = 2i\omega \underline{r}} \quad -(7)$$

The acceleration is:

$$\underline{a}_{orbital} = \frac{\partial \underline{v}}{\partial t} + c\underline{\nabla}\, v_0 \quad -(8)$$

2) where:

$$\frac{\partial \underline{v}}{\partial t} = c \underline{\nabla} v_0 \quad — (9)$$

by antisymmetry. So:

$$\boxed{\underline{a}_{orbital} = -4\omega^2 \underline{r}} \quad — (10)$$

and

$$\underline{\nabla} v_0 = -\frac{2\omega^2 \underline{r}}{c} \quad — (11)$$

From eq. (6) there are eqns for r_0 as follows:

$$\frac{\partial r_0}{\partial X} = -\frac{i\kappa r}{\sqrt{2}} \exp\left(i(\omega t - \kappa Z)\right) \quad — (12)$$

$$\frac{\partial r_0}{\partial Y} = \frac{\kappa r}{\sqrt{2}} \exp\left(i(\omega t - \kappa Z)\right) \quad — (13)$$

From eq. (11)

$$\frac{\partial v_0}{\partial X} = -\frac{2\omega^2}{\sqrt{2}c} \exp\left(i(\omega t - \kappa Z)\right) \quad — (14)$$

$$\frac{\partial v_0}{\partial Y} = -\frac{2i\omega^2}{\sqrt{2}c} \exp\left(i(\omega t - \kappa Z)\right). \quad — (15)$$

Here

$$r^\mu = (r_0, \underline{r}) \quad — (16)$$

$$v^\mu = (v_0, \underline{v}) \quad — (17)$$

Here r_0 and v_0 are interpreted using the Stokes Theorem as follows:

3)

$$r_0 = \frac{1}{c}\oint \underline{v} \cdot d\underline{r} = \frac{1}{c}\int_S \underline{\nabla} \times \underline{v} \cdot d\underline{A} \quad -(18)$$

and
$$v_0 = \frac{1}{c}\oint \underline{a} \cdot d\underline{r} = \frac{1}{c}\int_S \underline{\nabla} \times \underline{a} \cdot d\underline{A} \quad -(19)$$

where
$$\underline{a} = 2\frac{\partial \underline{v}}{\partial t} \quad -(20)$$

so
$$v_0 = 2\frac{\partial r_0}{\partial t} \quad -(21)$$

If the four momentum is defined as

$$p^\mu = m V^\mu = \left(\frac{E}{c}, \underline{p}\right) \quad -(22)$$

then
$$\boxed{E = m c v_0} \quad -(23)$$

therefore
$$E = 2mc\frac{\partial r_0}{\partial t} \quad -(24)$$

The rest energy for a photon is defined by de Broglie's Theorem:

$$E_0 = \hbar\omega = mc^2 \quad -(25)$$

in which case:
$$\boxed{c = 2\frac{\partial r_0}{\partial t}} \quad -(26)$$

so
$$r_0 \sim 1.5 \times 10^8 \text{ metres}$$

and is a rest length.